

RAY OPTICS

INTRODUCTION

The branch of Physics called optics deals with the behavior of light and other electromagnetic waves. Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomenon is known as geometric optics.

PROPAGATION OF LIGHT

Light travels along straight line in a medium or in vacuum. The path of light changes only when there is an object in its path or where the medium changes. We call this rectilinear (straight-line) propagation of light. Light that starts from a point A and passes through another point B in the same medium actually passes through all the points on the straight line AB. Such a straight line path of light is called a ray of light. Light rays start from each point of a source and travel along straight lines till they fall on an object or a surface separating two media (mediums). A bundle of light rays is called a beam of light.

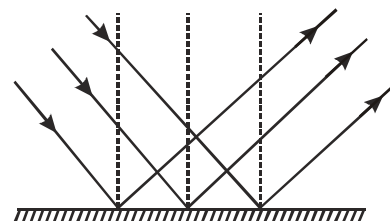
- ⊙ Apart from vacuum and gases, light can travel through some liquids and solids. A medium in which light can travel freely over large distances is called a transparent medium. Water, glycerine, glass and clear plastics are transparent. A medium in which light cannot travel is called opaque. Wood, metals, bricks, etc., are opaque. In materials like oil, light can travel some distance, but its intensity reduces rapidly. Such materials are called translucent.

REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is bounced back into the same medium. This is called **Reflection of light**.

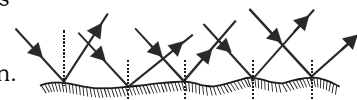
(i) Regular / Specular reflection :

When the reflection takes place from a perfect plane surface then after reflection rays remain parallel. It is called **Regular reflection**.



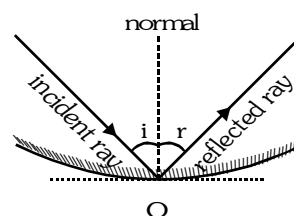
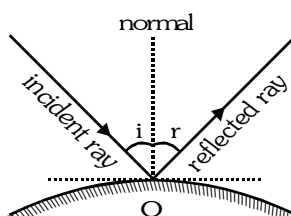
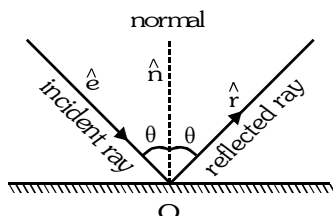
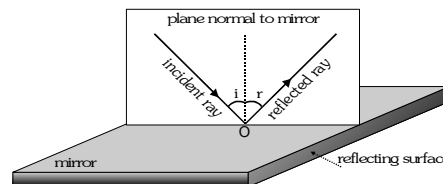
(ii) Diffused reflection

When the surface is rough, light is reflected from the surface from bits of its plane surfaces in irregular directions. This is called **diffused reflection**. This process enables us to see an object from any position.



LAWS OF REFLECTION

- ⊙ Incident ray, reflected ray and normal lies in the same plane.
- ⊙ The angle of reflection is equal to the angle of incident i.e. $\angle i = \angle r$.



In vector form

$$\vec{r} = \vec{e} - 2(\vec{e} \cdot \vec{n})\vec{n}$$

- **Rectilinear propagation of light**
- When a ray is incident on a surface, it is reflected back into the same medium.

- The frequency of light remains constant.

- Eye is not a source of light.

Due to

- Colour of light

- Speed of light

REFLECTION

Plane mirror



The

is

- ☉ To see complete image in a plane mirror the minimum length of plane mirror should be half the height of a person.

From figure. $\triangle HNM$ and $\triangle ENM$ are congruent

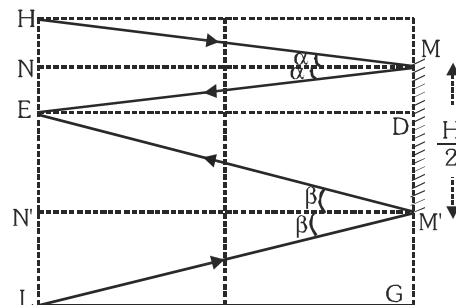
$$\therefore EN = HN \quad \therefore MD = EN = \frac{1}{2} HE$$

Similarly $\triangle EN' M'$ and $\triangle LN' M'$ are congruent

$$\begin{aligned} \text{Length of the mirror } MM' &= MD + M'D = \frac{1}{2} HE + \frac{1}{2} EL \\ &= \frac{1}{2} (HE + EL) = \frac{1}{2} HL \end{aligned}$$

\therefore Minimum of length of mirror is just half of the person.

- This result does not depend on position of eye (height of the eye from ground).
- This result is independent of distance of person in front of mirror.



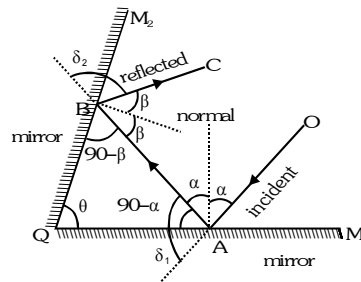
- ☉ Deviation for a single mirror  $\delta = 180 - (i + r)$; $\angle i = \angle r$; $\delta = 180 - 2i$

- ☉ Total deviation produced by the combination of two plane mirrors which are inclined at an angle θ from each other.

$$\delta = \delta_1 + \delta_2 = 180 - 2\alpha + 180 - 2\beta = 360 - 2(\alpha + \beta) \dots(i)$$

$$\text{From } \triangle QAB, \theta + 90 - \alpha + 90 - \beta = 180 \Rightarrow \theta = \alpha + \beta \dots(ii)$$

Putting the value of θ in (i) from (ii), $\delta = 360 - 2\theta$



- ☉ If there are two plane mirror inclined to each other at an angle θ the number of image (n) of a point object formed are determined as follows.

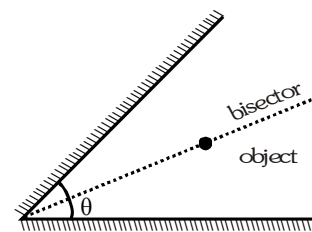
(a) If $\frac{360^\circ}{\theta} = m$ is even then number of images $n = m - 1$

(b) If $\frac{360^\circ}{\theta} = m$ is odd. There will be two case.

(i) When object is not on bisector, then number of images $n = m$

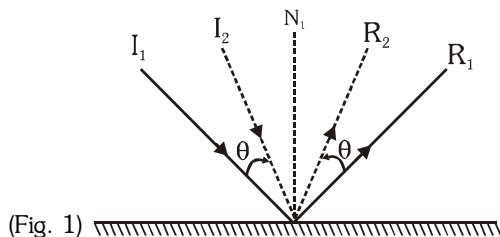
(ii) When object is at bisector, then number of images $n = m - 1$

(c) If $\frac{360^\circ}{\theta} = m$ is a fraction, and the object is placed symmetrically then no. of images $n = \text{nearest even integer}$

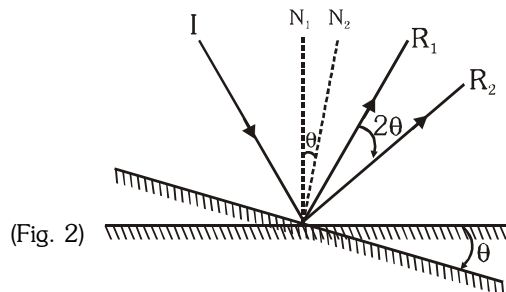


S.No.	θ in degree	$m = \frac{360^\circ}{\theta}$	Number of images formed if object is placed	
			asymmetrically	symmetrically
1.	0	∞	∞	∞
2.	30	12	11	11
3.	45	8	7	7
4.	60	6	5	5
5.	72	5	5	4
6.	75	4.8	–	4
7.	90	4	3	3
8.	112.5	3.2	–	4
9.	120	3	3	2

- ⊙ If the object is placed between two plane mirrors then images are formed due to multiple reflections. At each reflection, a part of light energy is absorbed. Therefore, distant images get fainter.
- ⊙ Keeping the mirror fixed if the incident ray is rotated by some angle, the reflected ray is also rotated by the same angle but in opposite sense. (See Fig. 1)

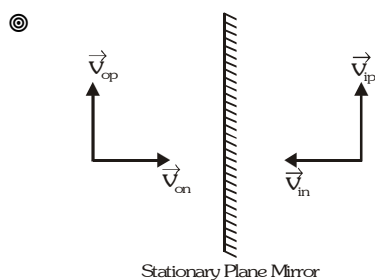


(Fig. 1)



(Fig. 2)

- ⊙ Keeping the incident ray fixed, if the mirror is rotated by some angle, then the reflected ray rotates by double the angle in the same sense. (See Fig. 2)



Stationary Plane Mirror

$$\vec{v}_{on} = -\vec{v}_{in}, \vec{v}_{op} = \vec{v}_{ip}$$

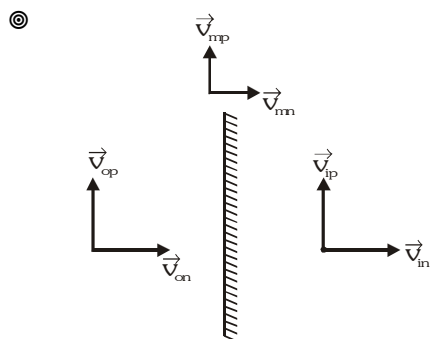
though speed of object and image are the same

v_{op} = component of velocity of object along parallel to mirror.

v_{on} = component of velocity of object along normal to mirror.

v_{ip} = component of velocity of image along parallel to mirror.

v_{in} = component of velocity of image along normal to mirror.



If mirror is moving $\vec{v}_{ip} = \vec{v}_{op}$ and $(\vec{v}_{im})_n = -(\vec{v}_{om})_n$

$$\Rightarrow \vec{v}_{in} - \vec{v}_{mn} = -(\vec{v}_{on} - \vec{v}_{mn}) \Rightarrow \vec{v}_{in} = 2\vec{v}_{mn} - \vec{v}_{on}$$

\vec{v}_{mn} = component of velocity of mirror along normal.

\vec{v}_{op} = component of velocity of object along mirror.

\vec{v}_{on} = component of velocity of object along normal

\vec{v}_{ip} = component of velocity of image along mirror.

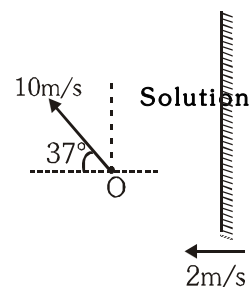
\vec{v}_{in} = component of velocity of image along normal.

Example

Find the velocity of the image.

$$\vec{v}_{ox} = (-10 \cos 37^\circ) \vec{i} = -8\vec{i} \text{ and } \vec{v}_{oy} = (10 \sin 37^\circ) \vec{j} = 6\vec{j}$$

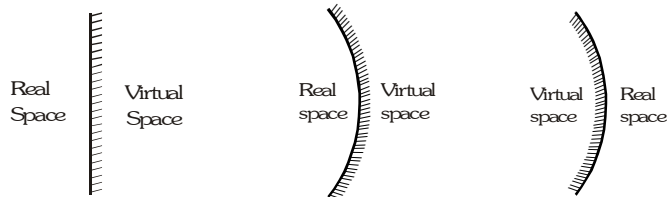
$$\vec{v}_{ix} = 2\vec{v}_{mx} - \vec{v}_{ox} = 2(-2\vec{i}) - (-8\vec{i}) = 4\vec{i} \text{ and } \vec{v}_{iy} = \vec{v}_{oy} = 6\vec{j}$$



REAL AND VIRTUAL SPACES

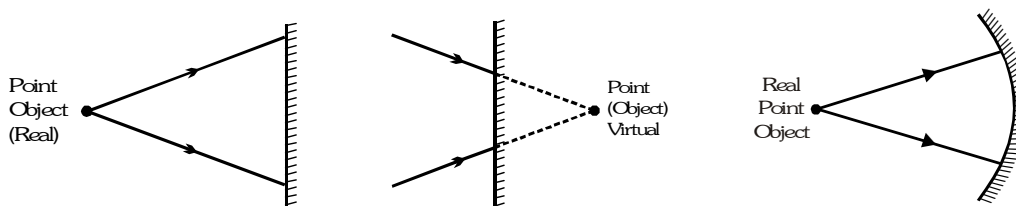
A mirror, plane or spherical divides the space into two ;

- Real space, a side where the reflected rays exist.
- Virtual space is on the other side where the reflected rays do not exist.



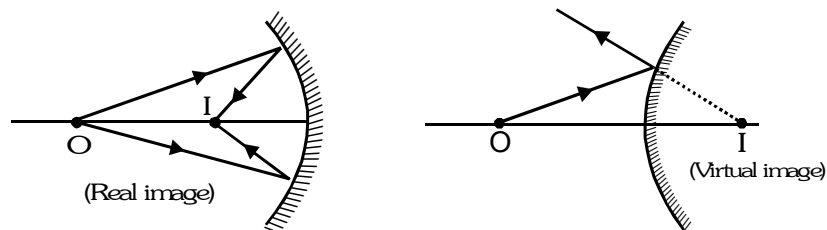
OBJECT

Object is decided by incident rays only. The point object is that point from which the incident rays actually diverge (Real object) or towards which the incident rays appear to converge (virtual object).



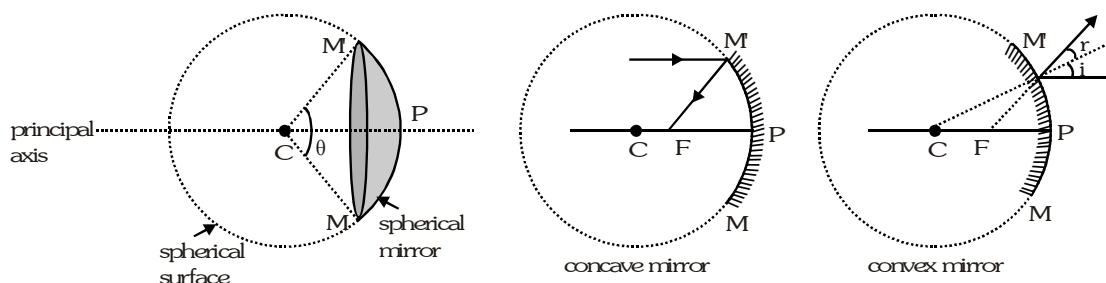
IMAGE

Image is decided by reflected or refracted rays only. The point image is that point at which the refracted / reflected rays reflected from the mirror, actually converge (real image) or from which the refracted /reflected rays appear to diverge (virtual image).



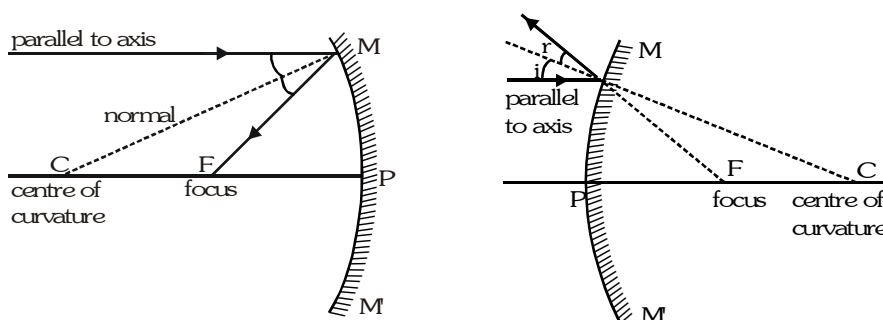
SPHERICAL (CURVED) MIRROR

Curved mirror is part of a hollow sphere. If reflection takes place from the inner surface then the mirror is called concave and if its outer surface acts as reflector it is convex.

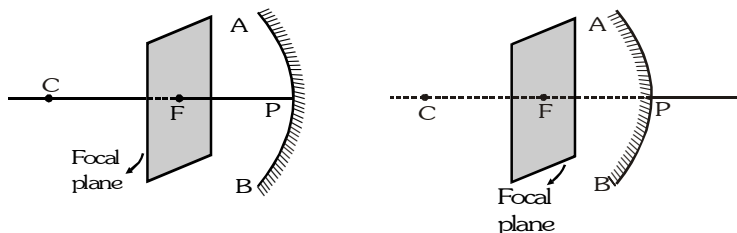


DEFINITIONS FOR THIN SPHERICAL MIRRORS

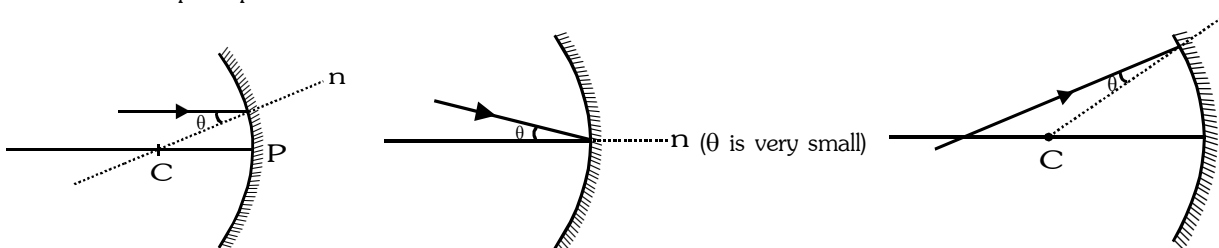
- (i) **Pole** is any point on the reflecting surface of the mirror. For convenience we take it as the midpoint P of the mirror (as shown).
- (ii) **Principal-section** is any section of the mirror such as MM' passing through the pole is called principal-section.
- (iii) **Centre of curvature** is the centre C of the sphere of which the mirror is a part.
- (iv) **Radius of curvature** is the radius R of the sphere of which the mirror is a part.
- (v) **Principal-axis** is the line CP, joining the pole and centre of curvature of the mirror.
- (vi) **Principal-focus** is an image point F on principal axis for which object is at infinity.



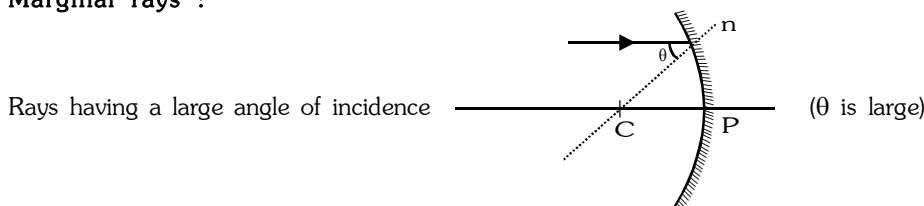
- (vii) **Focal-length** is the distance PF between pole P and focus F along principal axis.
- (viii) **Aperture**, in reference to a mirror, means the effective diameter of the light reflecting area of the mirror.
- (ix) **Focal Plane** is the plane passing through focus and perpendicular to principal axis.



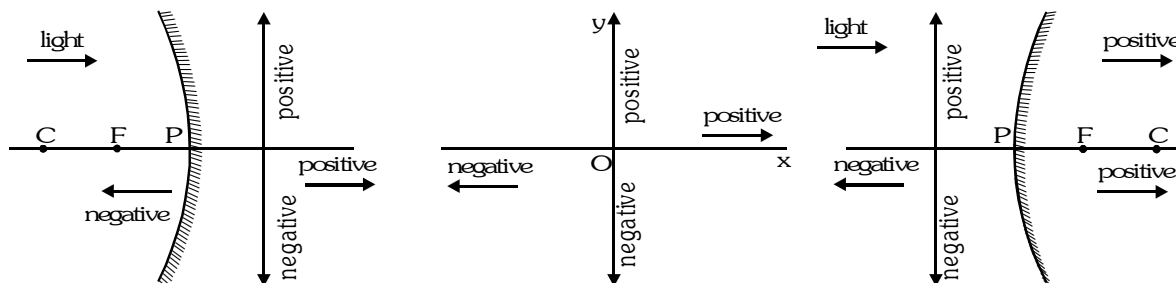
- (x) **Paraxial Rays** Those rays which make small angle with normal at point of incidence and hence are close to principal axis.



- (xi) **Marginal rays :**



SIGN- CONVENTION

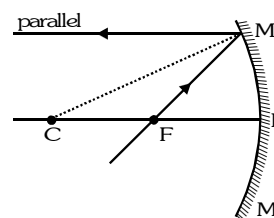
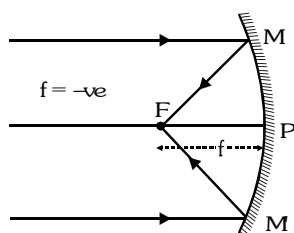


- Along principal axis, distances are measured from the pole (pole is taken as the origin).
- Distance in the direction of light are taken to be positive while opposite to be negative.
- The distances above principal axis are taken to be positive while below it negative.
- Whenever and wherever possible the ray of light is taken to travel from left to right.

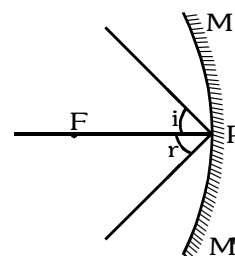
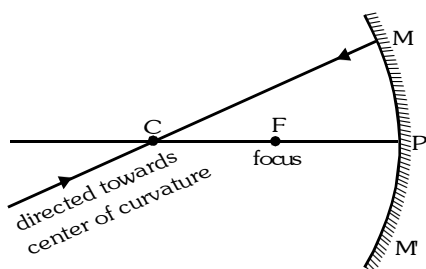
RULES FOR IMAGE FORMATION (FOR PARAXIAL RAYS ONLY)

(These rules are based on the laws of reflection $\angle i = \angle r$)

- | | |
|--|---|
| <p>⊙ A ray parallel to principal axis after reflection from the mirror passes or appears to pass through its focus (by definition of focus).</p> | <p>⊙ A ray passing through or directed towards focus, after reflection from the mirror, becomes parallel to the principal axis.</p> |
|--|---|



- | | |
|---|---|
| <p>⊙ A ray passing through or directed towards centre of curvature, after reflection from the mirror, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$).</p> | <p>⊙ Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis $\angle i = \angle r$.</p> |
|---|---|



RELATIONS FOR SPHERICAL MIRRORS

Relation between f and R for the spherical mirror

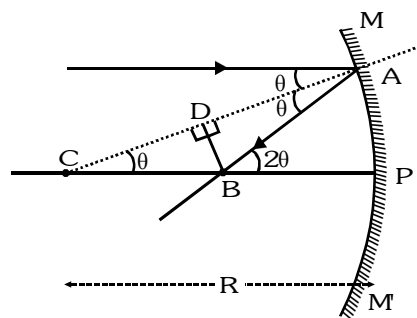
⊙ **For Marginal rays**

In $\triangle ABC$, $AB = BC$

$$AC = CD + DA = 2BC \cos \theta \Rightarrow R = 2BC \cos \theta$$

$$\Rightarrow BC = \frac{R}{2 \cos \theta} \text{ and } BP = PC - BC = R - \frac{R}{2 \cos \theta}$$

Note: B is not the focus ; it is just a point where a marginal ray after reflection meets.



⊙ **For paraxial rays (parallel to principal axis)**

$$(\theta \text{ small so } \sin \theta \approx \theta, \cos \theta \approx 1, \tan \theta \approx \theta) . \text{ Hence } BC = \frac{R}{2} \text{ and } BP = \frac{R}{2}$$

Thus, point B is the midpoint of PC (i.e. radius of curvature) and is defined as FOCUS so $BP = f = \frac{R}{2}$

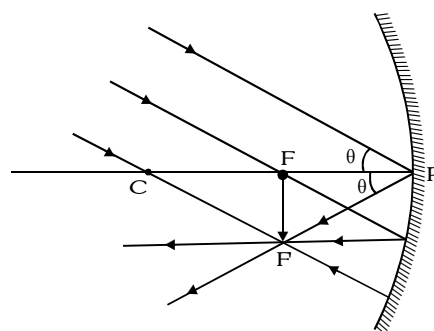
(Definition : Paraxial rays parallel to the principal axis after reflection from the mirror meet the principal axis at focus)

⊙ BP is the focal length (f) $\boxed{f = \frac{R}{2}}$

⊙ **Paraxial rays (not parallel to principal axis)**

Such rays after reflection meet at a point in the focal plane (F'), such that

$$\frac{FF'}{FP} = \tan \theta \approx \theta \Rightarrow \frac{FF'}{f} = \theta \Rightarrow FF' = f\theta$$



Relation between u, v and f for curved mirror

An object is placed at a distance u from the pole of a mirror and its image is formed at a distance v (from the pole)

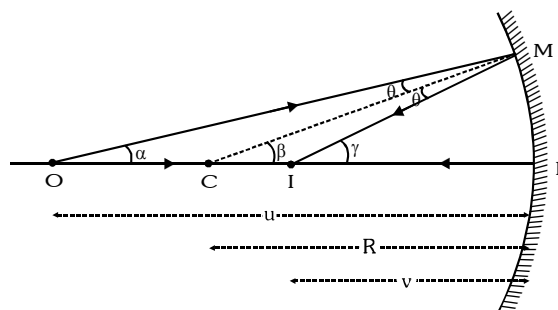
If angle is very small : $\alpha = \frac{MP}{u}, \beta = \frac{MP}{R}, \gamma = \frac{MP}{v}$

from $\triangle CMO$, $\beta = \alpha + \theta \Rightarrow \theta = \beta - \alpha$

from $\triangle CMI$, $\gamma = \beta + \theta \Rightarrow \theta = \gamma - \beta$

so we can write $\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \gamma + \alpha$

$$\therefore \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$



Sign convention for object/image for spherical mirrors

Real object	$u - ve$	Real image	$v - ve$
Virtual object	$u + ve$	Virtual image	$v + ve$

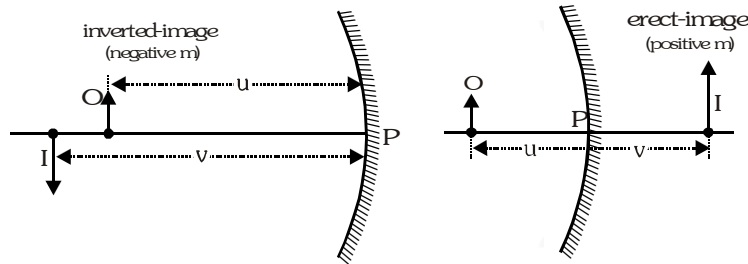
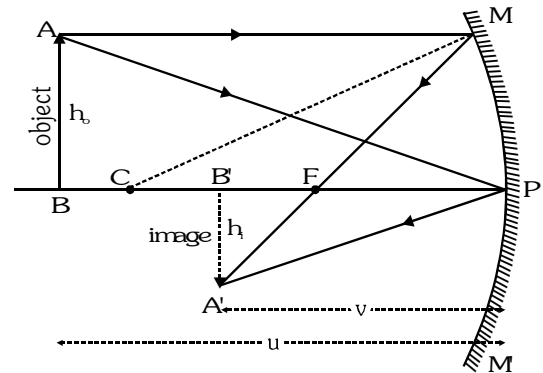
MAGNIFICATION

Transverse or lateral magnification

Linear magnification $m = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_o}$

ΔABP and $\Delta A'B'P$ are similar so $\frac{-h_i}{h_o} = \frac{-v}{-u} \Rightarrow \frac{h_i}{h_o} = -\frac{v}{u}$

Magnification $m = -\frac{v}{u}$; $m = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f} = \frac{h_i}{h_o}$



If one dimensional object is placed perpendicular to the principal axis then linear magnification is called transverse

or lateral magnification.

$$m = \frac{h_i}{h_o} = -\frac{v}{u}$$

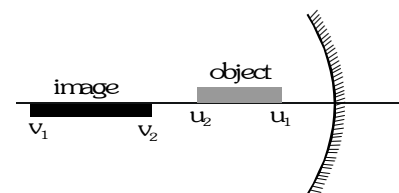
Magnification	Image	Magnification	Image
$ m > 1$	enlarged	$ m < 1$	diminished
$m < 0$	inverted	$m > 0$	erect

Longitudinal magnification

If one dimensional object is placed with its length along the principal axis then linear magnification is called longitudinal magnification.

Longitudinal magnification: $m_L = \frac{\text{length of image}}{\text{length of object}} = \left| \frac{v_2 - v_1}{u_2 - u_1} \right|$

For small objects only : $m_L = -\frac{dv}{du}$



differentiation of $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ gives us $-\frac{dv}{v^2} - \frac{du}{u^2} = 0 \Rightarrow -\frac{dv}{du} = \left[\frac{v}{u} \right]^2$ so $m_L = -\frac{dv}{du} = \left[\frac{v}{u} \right]^2 = m^2$

Superficial magnification

If two dimensional object placed with its plane perpendicular to principal axis its magnification is known as superficial magnification

Linear magnification $m = \frac{h_i}{h_o} = \frac{w_i}{w_o}$

$h_i = mh_o$, $w_i = mw_o$ and $A_{\text{obj}} = h_o \cdot w_o$

Area of image : $A_{\text{image}} = h_i \cdot w_i = mh_o \cdot mw_o = m^2 A_{\text{obj}}$

Superficial magnification $m_s = \frac{\text{area of image}}{\text{area of object}} = \frac{(ma) \times (mb)}{(a \times b)} = m^2$

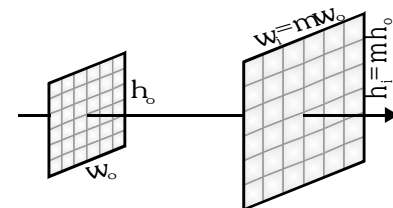


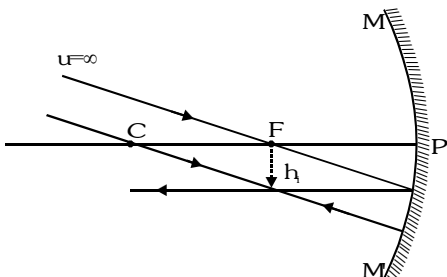
IMAGE FORMATION BY SPHERICAL MIRRORS

Concave mirror

(i) **Object** : Placed at infinity

Image : real, inverted, diminished at F

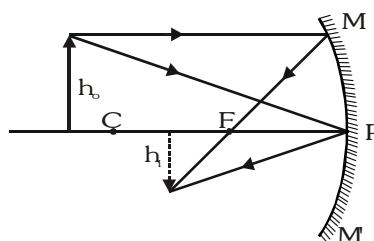
$$|m| \ll 1 \text{ \& } m < 0$$



(ii) **Object** : Placed in between infinity and C

Image : real, inverted, diminished in between C and F

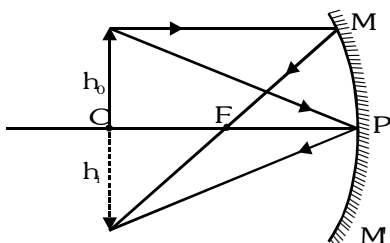
$$|m| < 1 \text{ \& } m < 0$$



(iii) **Object** : Placed at C

Image : real, inverted, equal at C

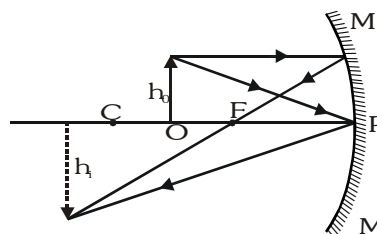
$$(m = -1)$$



(iv) **Object** : Placed in between F and C

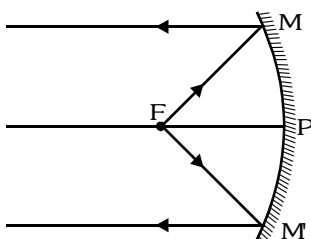
Image : real, inverted, enlarged beyond C

$$|m| > 1 \text{ \& } m < 0$$



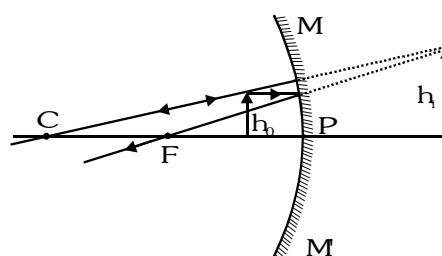
(v) **Object** : Placed at F

Image : real, inverted, very large (assumed) at infinity ($m \ll -1$)



(vi) **Object** : Placed between F and P

Image : virtual, erect, enlarged and behind the mirror ($m > +1$)

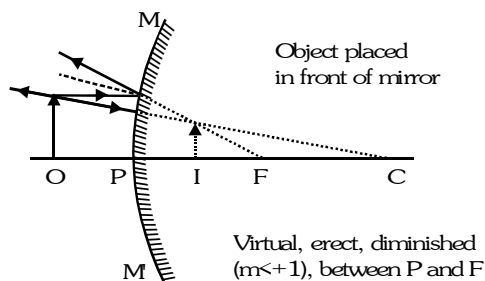
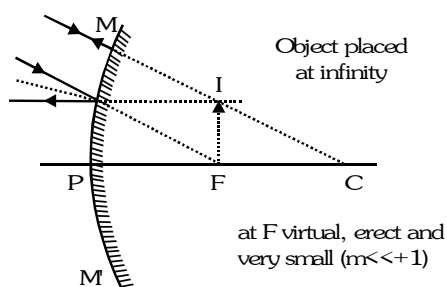


For concave mirror

Object	Image	Magnification
$-\infty$	F	$ m \ll 1 \text{ \& } m < 0$
$-\infty - C$	C - F	$ m < 1 \text{ \& } m < 0$
C	C	$m = -1$
C - F	$-\infty - C$	$ m > 1 \text{ \& } m < 0$
Just before F towards C	$-\infty$	$m \ll -1$
Just after F towards P	$+\infty$	$m \gg 1$

Convex mirror

Image is always virtual and erect, whatever be the position of the object and m is always positive.



GOLDEN KEY POINTS

- Differences in real & virtual image for spherical mirror

Real Image

- Inverted w.r.t. object
- Can be obtained on screen
- Its magnification is negative
- Forms in front of mirror

Virtual Image

- Erect w.r.t. object
- Can not be obtained on screen
- Its magnification is positive
- Forms behind the mirror

- For real extended object, if the image formed by a single mirror is erect it is always virtual (i.e., m is +ve) and in this situation if the size of image is :

Smaller than object the mirror is convex	Equal to object the mirror is plane	Larger than object the mirror is concave
$m < +1$ 	$m = +1$ 	$m > +1$

Example

The focal length of a concave mirror is 30cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

Solution

As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e., real) or erect (i.e., virtual), so there are two possibilities.

- (a) If the image is inverted (i.e., real)

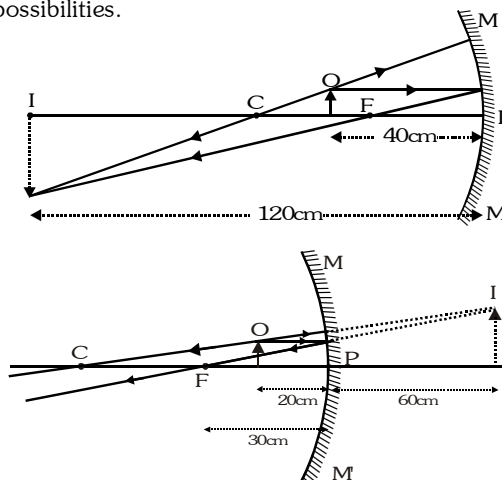
$$m = \frac{f}{f - u} \Rightarrow -3 = \frac{-30}{-30 - u} \Rightarrow u = -40 \text{ cm}$$

Object must be at a distance of 40 cm in front of the mirror (in between C and F).

- (b) If the image is erect (i.e., virtual)

$$m = \frac{f}{f - u} \Rightarrow 3 = \frac{-30}{-30 - u} \Rightarrow u = -20 \text{ cm}$$

Object must be at a distance of 20 cm in front of the mirror (in between F and P).



Example

A thin rod of length $\frac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. What is magnification?

Solution

Image is real and enlarged, the object must be between C and F.

One end A' of the image coincides with the end A of rod itself.

$$\text{So } v_A = u_A, \frac{1}{v_A} + \frac{1}{u_A} = \frac{1}{-f} \text{ i.e., } v_A = u_A = -2f$$

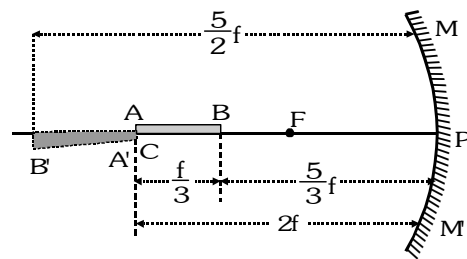
so it clear that the end A is at C. \therefore the length of rod is $\frac{f}{3}$

$$\therefore \text{Distance of the other end B from P is } u_B = 2f - \frac{f}{3} = \frac{5}{3}f$$

$$\text{if the distance of image of end B from P is } v_B \text{ then } \frac{1}{v_B} + \frac{1}{-\frac{5}{3}f} = \frac{1}{-f} \Rightarrow v_B = -\frac{5}{2}f$$

$$\therefore \text{the length of the image } |v_B| - |v_A| = \frac{5}{2}f - 2f = \frac{1}{2}f \text{ and magnification } m = \frac{|v_B| - |v_A|}{|u_B| - |u_A|} = \frac{\frac{1}{2}f}{-\frac{1}{3}f} = -\frac{3}{2}$$

Negative sign implies that image is inverted with respect to object and so it is real.

**Example**

A concave mirror of focal length 10 cm and convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirror on their common axis and 15 cm from the concave mirror. Find the position of image produced by the reflection first at concave mirror and then at convex mirror.

Solution

For M_1 mirror O act as a object, let its image is I_1 then,

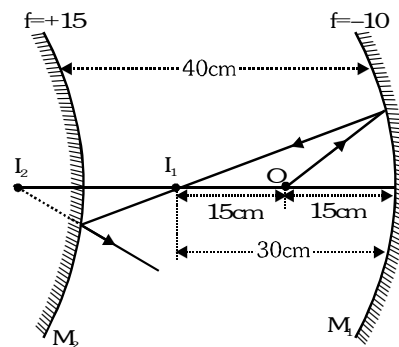
$$u = -15 \text{ cm, } f = -10 \text{ cm} \Rightarrow \frac{1}{v} + \frac{1}{-15} = \frac{1}{-10} \Rightarrow v = -30 \text{ cm}$$

Image I_1 will act as a object for mirror M_2 its distance from mirror M_2 .

$$u_1 = -(40 - 30) \text{ cm} = -10 \text{ cm}$$

$$\text{so } \frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1}{v_1} + \frac{1}{-10} = \frac{1}{15} \Rightarrow v_1 = +6 \text{ cm}$$

So final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual.

**Example**

The sun subtends an angle θ radians at the pole of a concave mirror of focal length f . What is the diameter of the image of the sun formed by the mirror.

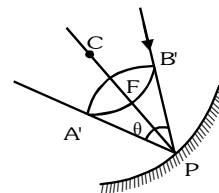
Solution

Since the sun is at large distance very distant, u is very large and so $\frac{1}{u} \approx 0$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{f} \Rightarrow v = -f$$

The image of sun will be formed at the focus and will be real, inverted and diminished

$$A'B' = \text{height of image and } \theta = \frac{\text{Arc}}{\text{Radius}} = \frac{A'B'}{FP} \Rightarrow \theta = \frac{d}{f} \Rightarrow d = f\theta$$



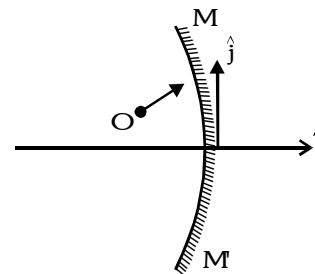
VELOCITY OF IMAGE OF MOVING OBJECT (SPHERICAL MIRROR)

(a) Velocity component along axis (Longitudinal velocity)

When an object is coming from infinite towards the focus of concave mirror

$$\therefore -\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0 \Rightarrow \vec{v}_{ix} = -\frac{v^2}{u^2} \vec{v}_{ox} = -m^2 \vec{v}_{ox}$$

$$\left[v_{ix} = \frac{dv}{dt} = \text{velocity of image along principal-axis; } v_{ox} = \frac{du}{dt} = \text{velocity of object along principal-axis} \right]$$



(b) Velocity component perpendicular to axis (Transverse velocity)

$$m = \frac{h_i}{h_o} = -\frac{v}{u} = \frac{f}{f-u} \Rightarrow h_i = \left(\frac{f}{f-u} \right) h_o$$

$$\frac{dh_i}{dt} = \left(\frac{f}{f-u} \right) \frac{dh_o}{dt} + \frac{f h_o}{(f-u)^2} \frac{du}{dt} ; \vec{v}_{iy} = \left[m \vec{v}_{oy} + \frac{m^2 h_o}{f} \vec{v}_{ox} \right] \tilde{j}$$

$$\left[\begin{array}{l} \frac{dh_i}{dt} = \text{velocity of image } \perp \text{ to principal-axis} \\ \frac{dh_o}{dt} = \text{velocity of object } \perp \text{ to principal-axis} \end{array} \right]$$

Note : Here principal axis has been taken to be along x-axis.

POWER OF A MIRROR

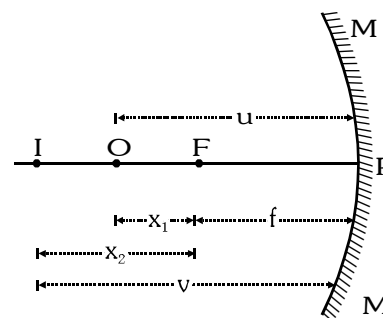
The power of a mirror is defined as $P = -\frac{1}{f(m)} = -\frac{100}{f(cm)}$

NEWTON'S FORMULA

In case of spherical mirrors if object distance (x_1) and image distance (x_2) are measured from focus instead of pole, $u = -(f+x_1)$ and $v = -(f+x_2)$,

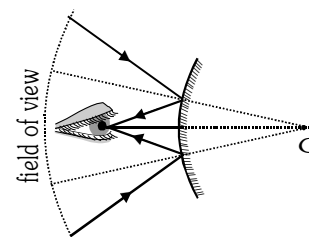
$$\text{by } \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow -\frac{1}{(f+x_2)} - \frac{1}{(f+x_1)} = -\frac{1}{f}$$

on solving $x_1 x_2 = f^2$ This is Newton's formula.



GOLDEN KEY POINTS

- Convex mirrors** give erect, virtual and diminished image. In convex mirror the field of view is increased as compared to plane mirror. It is used as rear-view mirror in vehicles.
- Concave mirrors** give enlarged, erect and virtual image, so these are used by dentists for examining teeth. Due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights.
- As focal length of a spherical mirror $f=R/2$ depends only on the radius of mirror and is independent of wavelength of light and refractive index of medium so the focal length of a spherical mirror in air or water and for red or blue light is same.



REFRACTION

Refraction is the phenomenon in which direction of propagation of light changes at the boundary when it passes from one medium to the other. In case of refraction frequency does not change.

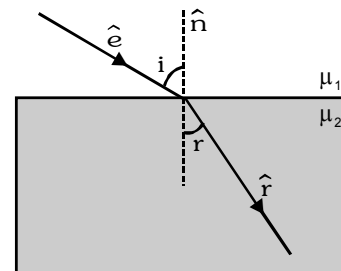
⊙ Laws of Refraction

- (i) Incident ray, refracted ray and normal always lie in the same plane .

In vector form $(\vec{e} \times \vec{n}) \cdot \vec{r} = 0$

- (ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

In vector form $\mu_1 |\vec{e} \cdot \vec{n}| = \mu_2 |\vec{r} \cdot \vec{n}|$



Absolute refractive index

It is defined as the ratio of speed of light in free space 'c' to that in a given medium v. μ or $n = \frac{c}{v}$

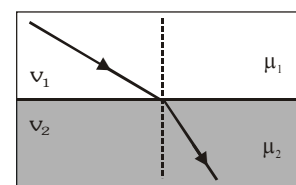
Denser is the medium, lesser will be the speed of light and so greater will be the refractive index,

$$\because v_{\text{glass}} < v_{\text{water}}, \quad \therefore \mu_{\text{G}} > \mu_{\text{W}}$$

Relative refractive index

When light passes from one medium to the other, the refractive index of medium 2 relative to 1 is written as ${}_1\mu_2$ and is defined as

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{(c/v_2)}{(c/v_1)} = \frac{v_1}{v_2}$$



⊙ Bending of light ray

According to Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

- (i) If light passes from rarer to denser medium $\mu_1 = \mu_R$ and $\mu_2 = \mu_D$

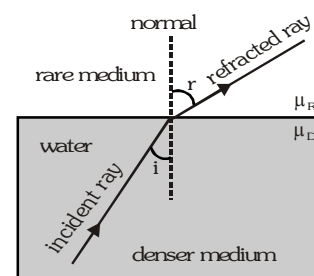
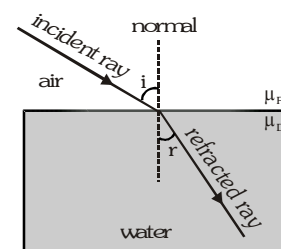
$$\text{so that } \frac{\sin i}{\sin r} = \frac{\mu_D}{\mu_R} > 1 \Rightarrow \angle i > \angle r$$

In passing from rarer to denser medium, the ray bends towards the normal.

- (ii) If light passes from denser to rarer medium $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$

$$\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D} < 1 \Rightarrow \angle i < \angle r$$

In passing from denser to rarer medium, the ray bends away from the normal.



APPARENT DEPTH AND NORMAL SHIFT

If a point object in denser medium is observed from rarer medium and boundary is plane, then from Snell's law we have $\mu_D \sin i = \mu_R \sin r$...(i)
If the rays OA and OB are close enough to reach the eye.

$$\sin i \approx \tan i = \frac{p}{d_{ac}} \quad \text{and} \quad \sin r \approx \tan r = \frac{p}{d_{ap}}$$

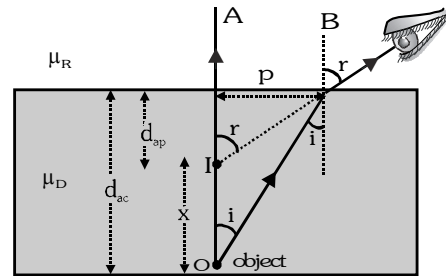
here d_{ac} = actual depth, d_{ap} = apparent depth

$$\text{So that equation (i) becomes } \mu_D = \frac{p}{d_{ac}} = \mu_R \frac{p}{d_{ap}} \Rightarrow \frac{d_{ac}}{d_{ap}} = \frac{\mu_D}{\mu_R} = \frac{\mu_1}{\mu_2}$$

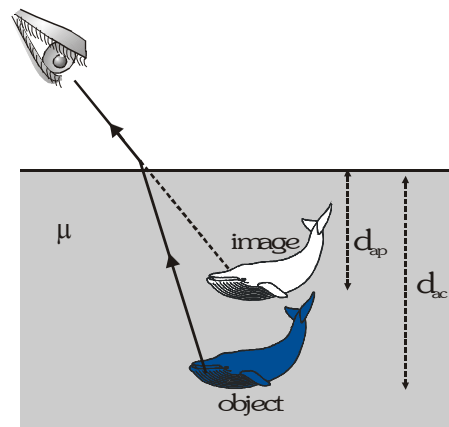
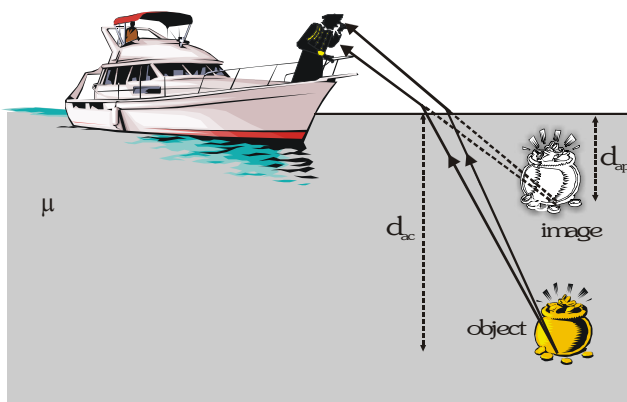
$$(\text{If } \mu_R = 1, \mu_D = \mu) \text{ then } d_{ap} = \frac{d_{ac}}{\mu} \quad \text{so } d_{ap} < d_{ac} \dots \text{(ii)}$$

The distance between object and its image, called normal shift (x)

$$x = d_{ac} - d_{ap} \left[\because d_{ap} = \frac{d_{ac}}{\mu} \right]; \quad x = d_{ac} - \frac{d_{ac}}{\mu} = d_{ac} \left[1 - \frac{1}{\mu} \right] \dots \text{(iii)}$$



$$\text{If } d_{ac} = d \text{ then } x = d \left[1 - \frac{1}{\mu} \right]$$



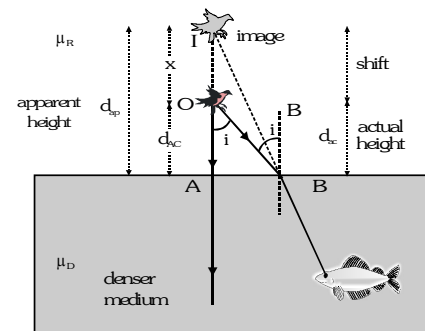
Object in a rarer medium is seen from a denser medium

$$\frac{d_{ac}}{d_{ap}} = \frac{\mu_1}{\mu_2} = \frac{\mu_R}{\mu_D} = \frac{1}{\mu} (< 1)$$

$$d_{ap} = \mu d_{ac} \quad \text{i.e., } d_{ap} > d_{ac}$$

A high flying object appears to be higher than in reality.

$$x = d_{ap} - d_{ac} \Rightarrow x = [\mu - 1] d_{ac}$$



LATERAL SHIFT

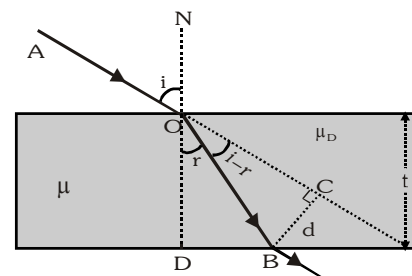
The perpendicular distance between incident and emergent ray is known as lateral shift.

Lateral shift $d = BC$ and t = thickness of slab

$$\text{In } \triangle BOC : \sin(i - r) = \frac{BC}{OB} = \frac{d}{OB} \Rightarrow d = OB \sin(i - r) \dots \text{(i)}$$

$$\text{In } \triangle OBD : \cos r = \frac{OD}{OB} = \frac{t}{OB} \Rightarrow OB = \frac{t}{\cos r} \dots \text{(ii)}$$

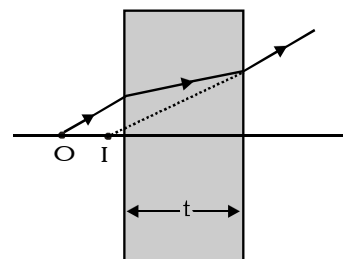
$$\text{From (i) and (ii) } d = \frac{t}{\cos r} \sin(i - r)$$



TRANSPARENT GLASS SLAB (Normal shift)

When an object is placed in front of a glass slab, it shifts the object in the direction of incident light and forms an image at a distance x .

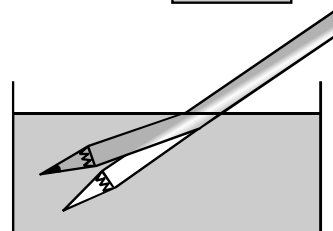
$$x = t \left[1 - \frac{1}{\mu} \right]$$

**SOME ILLUSTRATIONS OF REFRACTION**

⊙ **Bending of an object**

When a point object in a denser medium is seen from a

rarer medium it appears to bend by $\frac{d}{\mu}$



⊙ **Twinkling of stars**

Due to fluctuations in refractive index of atmosphere the refraction becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars.

GOLDEN KEY POINTS

- μ is a scalar and has no units and dimensions.
- If ϵ_0 and μ_0 are electric permittivity and magnetic permeability respectively of free space while ϵ and μ those of a given medium, then according to electromagnetic theory.

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \text{ and } v_m = \frac{1}{\sqrt{\epsilon \mu}} \Rightarrow n_m = \frac{c}{v} = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} = \sqrt{\epsilon_r \mu_r}$$

- As in vacuum or free space, speed of light of all wavelengths is maximum and equal to c so for all wavelengths the refractive index of free space is minimum and is $\mu = \frac{c}{v_m} = \frac{c}{c} = 1$

Example

A ray of light is incident on a transparent glass slab of refractive index 1.62. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence? [$\tan^{-1}(1.62) = 58.3^\circ$]

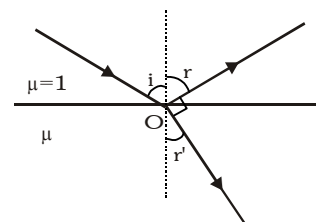
Solution

According to given problem : $r + 90^\circ + r' = 180^\circ$ i.e., $r' = 90^\circ - r$

$\Rightarrow r' = (90^\circ - i)$ [$\because \angle i = \angle r$] and as according to Snell's law: $1 \sin i = \mu \sin r'$

$\sin i = \mu \sin (90^\circ - i) \Rightarrow \sin i = \mu \cos i$ [$\because \sin (90^\circ - i) = \cos i$]

$\Rightarrow \tan i = \mu \Rightarrow i = \tan^{-1} \mu = \tan^{-1}(1.62) = 58.3^\circ$

**Example**

A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. An object is kept in air at a distance 40 cm from the mirror. Find the position of image w.r.t an observer near the object. What is effect of separation between glass slab and the mirror on image.

Solution

Shifting in object due to glass slab $x = d \left(1 - \frac{1}{\mu} \right) = 20 \left[1 - \frac{1}{1.5} \right] = \frac{20}{3}$ cm

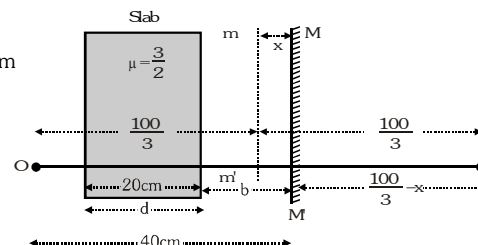
Distance of object from mirror (as seen by mirror) $= 40 - \frac{20}{3} = \frac{100}{3}$ cm

Image will be formed at a distance $\frac{100}{3}$ cm from mirror M.

Shifting in image due to glass slab $= \frac{20}{3}$ cm

So distance of image from mirror $= \frac{100}{3} - \frac{20}{3} = \frac{80}{3}$ cm

Distance of image from the actual plane mirror is independent of separation b between glass slab and the mirror. If the distance is more then brightness of image will be less.



Example

If one face of a prism angle 30° and $\mu = \sqrt{2}$ is silvered, the incident ray retraces its initial path. What is the angle of incidence ?

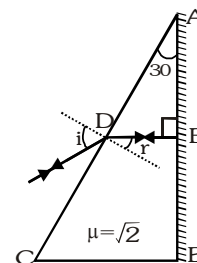
Solution

As incident ray retraces its path the ray is incident normally on the silvered face of the prism as shown in figure.

$$\text{Further, as in } \triangle AED \quad 30^\circ + 90^\circ + \angle D = 180^\circ \Rightarrow \angle D = 60^\circ$$

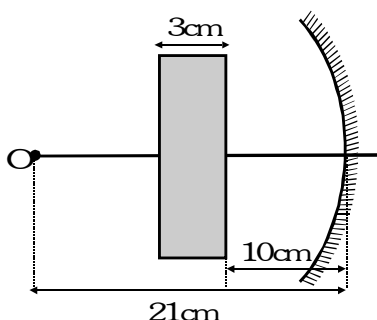
$$\text{Now as by construction, } \angle D + \angle r = 90^\circ \Rightarrow \angle r = 90^\circ - 60^\circ = 30^\circ$$

$$\therefore \text{ from Snell's law at surface AC, } 1 \sin i = \sqrt{2} \sin 30^\circ = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}} \therefore \sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45^\circ$$



Example

An object is placed 21 cm in front of a concave mirror of radius of curvature 20 cm. A glass slab of thickness 3 cm and refractive index 1.5 is placed closed to the mirror in space between the object and the mirror. Find the position of final image formed if distance of nearer surface of the slab from the mirror is 10 cm.



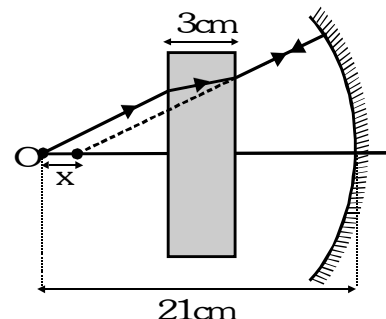
Solution

$$\text{Shift by slab } x = d \left(1 - \frac{1}{\mu} \right) = 3 \left(1 - \frac{1}{1.5} \right) = 1 \text{ cm}$$

$$\text{for image formed by mirror } u = - (21 - 1) \text{ cm} = - 20 \text{ cm.}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{-20} + \frac{1}{v} = \frac{1}{-10} \Rightarrow v = - 20 \text{ cm}$$

$$\text{shift in the direction of light } v = - (20 + 1) = - 21 \text{ cm.}$$



Example

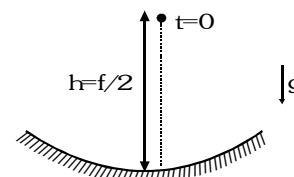
A particle is dropped along the axis from a height $\frac{f}{2}$ on a concave mirror of focal length f as shown in figure. Find the maximum speed of image.

Solution

$$v_{IM} = -m^2 v_{OM} = -m^2 (gt) \text{ where}$$

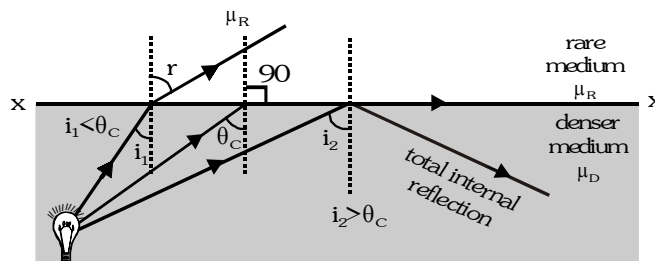
$$m = \frac{f}{f-u} = \frac{-f}{-f + \left(\frac{f}{2} - \frac{gt^2}{2} \right)} = \frac{2f}{f+gt^2} \Rightarrow v_1 = - \left(\frac{2f}{f+gt^2} \right)^2 (gt) = \frac{-4f^2 gt}{(f+gt^2)^2}$$

$$\text{For maximum speed } \frac{dv_1}{dt} = 0 \Rightarrow t = \sqrt{\frac{f}{3g}} \Rightarrow v_{\text{max}} = \frac{3}{4} \sqrt{3fg}$$



TOTAL INTERNAL REFLECTION

When light ray travel from denser to rarer medium it bend away from the normal if the angle of incident is increased, angle of refraction will also increased. At a particular value of angle the refracted ray subtend 90° angle with the normal, this angle of incident is known as critical angle (θ_c). If angle of incident further increase the ray come back in the same medium this phenomenon is known as total internal reflection.



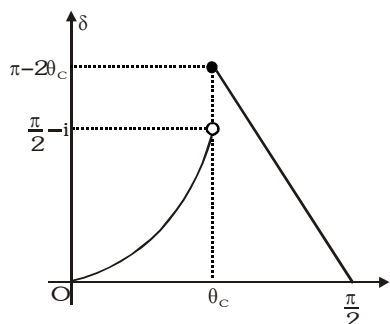
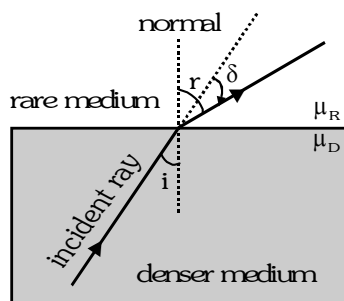
CONDITIONS

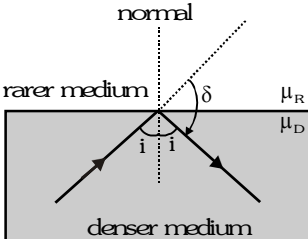
- Angle of incident $>$ critical angle [$i > \theta_c$]
- Light should travel from denser to rare medium \Rightarrow Glass to air, water to air, Glass to water

Snell's Law at boundary xx' , $\mu_D \sin \theta_c = \mu_R \sin 90^\circ \Rightarrow \sin \theta_c = \frac{\mu_R}{\mu_D}$

Graph between angle of deviation (δ) and angle of incidence (i) as rays goes from denser to rare medium

- If $i < \theta_c$, $\mu_D \sin i = \mu_R \sin r$; $r = \sin^{-1}\left(\frac{\mu_D}{\mu_R} \sin i\right)$ so $\delta = r - i = \sin^{-1}\left(\frac{\mu_D}{\mu_R} \sin i\right) - i$



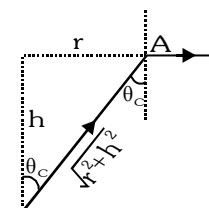
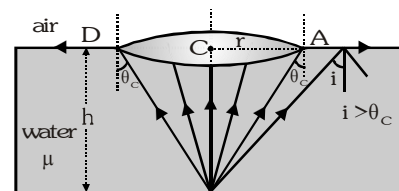
- If $i > \theta_c$; $\delta = \pi - 2i$
- 

- A point object is situated at the bottom of tank filled with a liquid of refractive index μ upto height h . It is found light from the source come out of liquid surface through a circular portion above the object

$$\sin \theta_c = \frac{r}{\sqrt{r^2 + h^2}} \quad \& \quad \sin \theta_c = \frac{1}{\mu} \Rightarrow \frac{1}{\mu} = \frac{r}{\sqrt{r^2 + h^2}} \Rightarrow \frac{1}{\mu^2} = \frac{r^2}{r^2 + h^2}$$

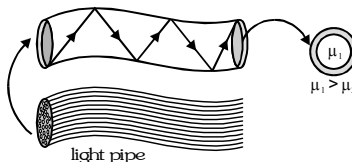
$$\Rightarrow \mu^2 r^2 = r^2 + h^2 \Rightarrow (\mu^2 - 1)r^2 = h^2 \Rightarrow \text{radius of circular portion}$$

$$r = \frac{h}{\sqrt{\mu^2 - 1}} \quad \text{and} \quad \text{area} = \pi r^2$$

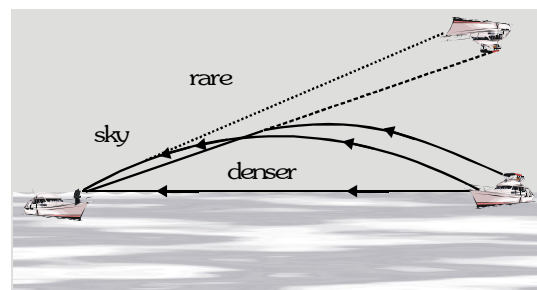
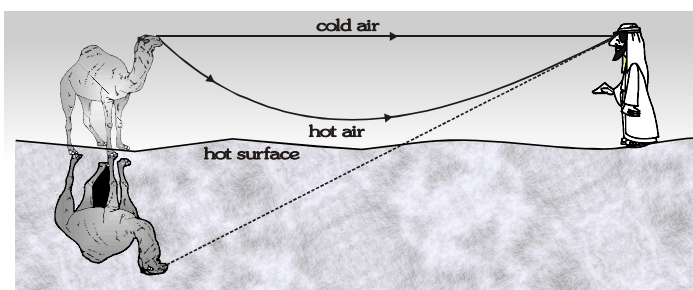


SOME ILLUSTRATIONS OF TOTAL INTERNAL REFLECTION

- ◎ **Sparkling of diamond** : The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so $\theta_c = 24^\circ$. Now the cutting of diamond are such that $i > \theta_c$. So TIR will take place again and again inside it. The light which beams out from a few places in some specific directions makes it sparkle.
- ◎ **Optical Fibre** : In it light through multiple total internal reflections is propagated along the axis of a glass fibre of radius of few microns in which index of refraction of core is greater than that of surroundings.



- ◎ **Mirage and looming** : Mirage is caused by total internal reflection in deserts where due to heating of the earth, refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with $i > \theta_c$ so that TIR will take place and we see the image of an object along with the object as shown in figure.



Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here μ decreases with height and so the image of an object is formed in air if ($i > \theta_c$) as shown in figure.

GOLDEN KEY POINTS

- A diver in water at a depth d sees the world outside through a horizontal circle of radius, $r = d \tan \theta_c$.
- In case of total internal reflection, as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirror or refraction from lenses there is some loss of intensity as all light can never be reflected or refracted. This is why images formed by TIR are much brighter than formed by mirrors or lenses.

Example

A rectangular block of glass is placed on a printed page laying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block.

Solution

The situation is depicted in figure. Light will not emerge out from the vertical face BC if at it

$$i > \theta_c \quad \text{or} \quad \sin i > \sin \theta_c \Rightarrow \sin i > \frac{1}{\mu} \quad \text{as } \sin \theta_c = \frac{1}{\mu} \quad \dots (i)$$

But from Snell's law at O $\sin \theta = \mu \sin r$

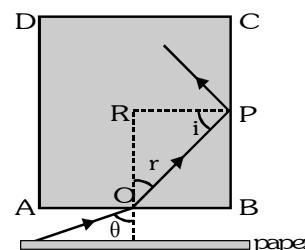
$$\text{And in } \triangle OPR, r + 90 + i = 180 \Rightarrow r + i = 90 \Rightarrow r = 90 - i$$

$$\text{So } \sin \theta = \mu \sin (90 - i) = \mu \cos i \Rightarrow \cos i = \frac{\sin \theta}{\mu}$$

$$\text{so } \sin i = \sqrt{1 - \cos^2 i} = \sqrt{1 - \left[\frac{\sin \theta}{\mu} \right]^2} \quad \dots (ii)$$

so substituting the value of $\sin i$ from equation (ii) in (i),

$$\sqrt{1 - \frac{\sin^2 \theta}{\mu^2}} > \frac{1}{\mu} \quad \text{i.e., } \mu^2 > 1 + \sin^2 \theta \because (\sin^2 \theta)_{\max} = 1 \therefore \mu^2 > 2 \Rightarrow \mu > \sqrt{2} \therefore \mu_{\min} = \sqrt{2}$$



REFRACTION AT TRANSPARENT CURVED SURFACE

μ_1 = refractive index of the medium in which actual incident ray lies.

μ_2 = refractive index of the medium in which actual refractive ray lies.

O = Object

P = pole

C = centre of curvature

R = PC = radius of curvature

Refraction from curved surface

$$\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$$

if angle is very small : $\mu_1 \theta_1 = \mu_2 \theta_2 \dots (i)$

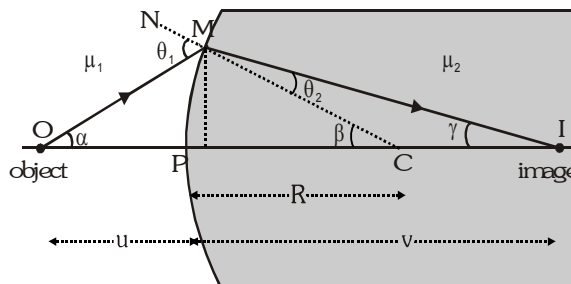
$$\text{But } \theta_1 = \alpha + \beta \dots (ii)$$

$$\beta = \theta_2 + \gamma \dots (iii)$$

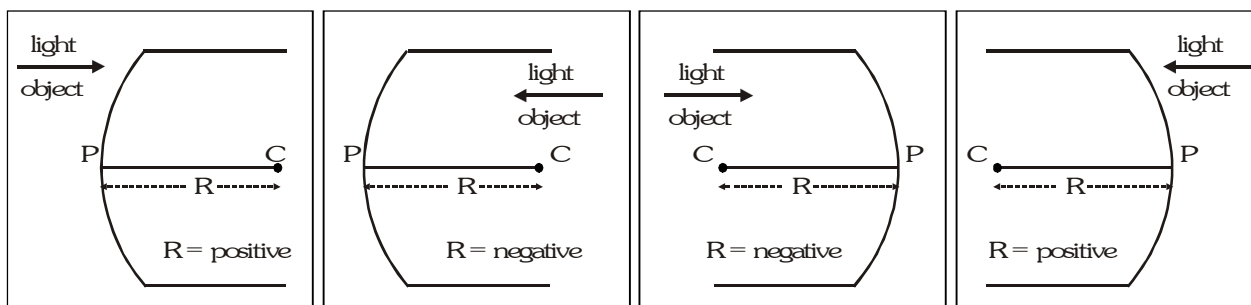
from (i), (ii) and (iii) $\mu_1(\alpha + \beta) = \mu_2(\beta + \gamma)$

$$\Rightarrow \mu_1 \alpha + \mu_1 \beta = \mu_2 \beta + \mu_2 \gamma \Rightarrow \mu_1 \alpha + \mu_2 \gamma = (\mu_2 - \mu_1) \beta$$

$$\Rightarrow \frac{\mu_1 PM}{-u} + \frac{\mu_2 PM}{v} = \frac{(\mu_2 - \mu_1) PM}{R} \Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$



SIGN CONVENTION FOR RADIUS OF CURVATURE



These are valid for all single refraction surfaces – convex, concave or plane. In case of plane refracting

$$\text{surface } R \rightarrow \infty, \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = 0 \text{ i.e. } \frac{u}{v} = \frac{\mu_1}{\mu_2} \text{ or } \frac{d_{Ac}}{d_{Ap}} = \frac{\mu_1}{\mu_2}$$

FOCAL LENGTH OF A SINGLE SPHERICAL SURFACE

A single spherical surface has two principal focus points which are as follows–

- (i) **First focus:** The first principal focus is the point on the axis where when an object is placed, the image is formed at infinity. That is when

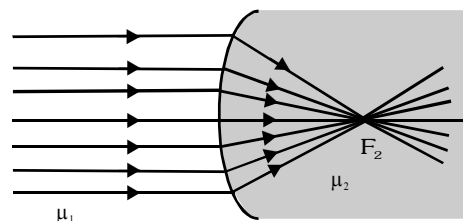
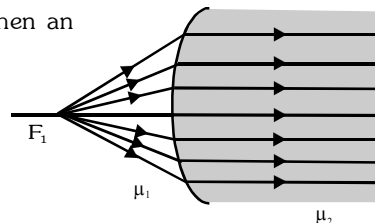
$$u = f_1, v = \infty, \text{ then from } -\frac{\mu_1}{u} + \frac{\mu_2}{v} = \left(\frac{\mu_2 - \mu_1}{R} \right)$$

$$\text{We get } -\frac{\mu_1}{f_1} = \frac{\mu_2 - \mu_1}{R} \Rightarrow f_1 = \frac{-\mu_1 R}{(\mu_2 - \mu_1)}$$

- (ii) **Second focus:** Similarly, the second principal focus is the point where parallel rays focus. That is $u_1 = -\infty, v_1 = f_2$, then

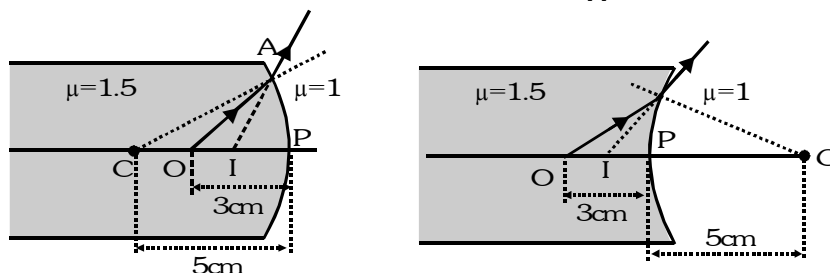
$$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R}; f_2 = \frac{\mu_2 R}{(\mu_2 - \mu_1)}$$

- (iii) **Ratio of Focal length:** $\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}$



Example

An air bubble in glass ($\mu = 1.5$) is situated at a distance 3 cm from a spherical surface of diameter 10 cm as shown in Figure. At what distance from the surface will the bubble appear if the surface is (a) convex (b) concave.



Solution

In case of refraction from curved surface $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

$$(a) \quad \mu_1 = 1.5, \quad \mu_2 = 1, \quad R = -5 \text{ cm and } u = -3 \text{ cm} \Rightarrow \frac{1}{v} - \frac{(1.5)}{(-3)} = \frac{1-1.5}{(-5)} \Rightarrow v = -2.5 \text{ cm}$$

the bubble will appear at a distance 2.5 cm from the convex curved surface inside the glass.

$$(b) \quad \mu_1 = 1.5, \quad \mu_2 = 1, \quad R = 5 \text{ cm and } u = -3 \text{ cm} \Rightarrow \frac{1}{v} - \frac{(1.5)}{(-3)} = \frac{1-1.5}{(5)} \Rightarrow v = -1.66 \text{ cm}$$

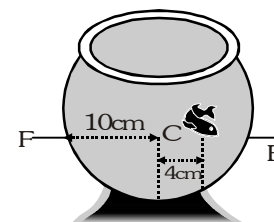
the bubble will appear at a distance 1.66 cm from the concave curved surface inside the glass.

Note : If the surface is plane then $R \rightarrow \infty$

$$\text{case (a) or (b) would yield } \frac{1}{v} - \frac{(1.5)}{(-3)} = \frac{(1-1.5)}{\infty} \Rightarrow v = -2 \text{ cm}$$

Example

In a thin spherical fish bowl of radius 10 cm filled with water of refractive index $(4/3)$, there is a small fish at a distance 4 cm from the centre C as shown in Figure. Where will the fish appear to be, if seen from (a) E and (b) F (neglect the thickness of glass) ?



Solution

In the case of refraction from curved surface $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

$$(a) \quad \text{Seen from E } \mu_1 = \frac{4}{3}, \quad \mu_2 = 1, \quad R = -10 \text{ cm \& } u = -(10 - 4) = -6 \text{ cm}$$

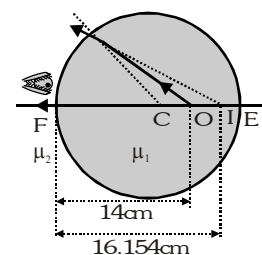
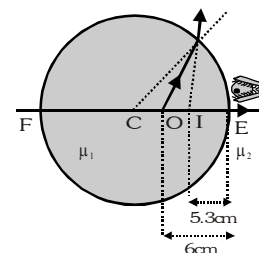
$$\Rightarrow \frac{1}{v} - \frac{\frac{4}{3}}{-6} = \frac{1 - \frac{4}{3}}{-10} \Rightarrow v = \frac{90}{17} = -5.3 \text{ cm}$$

i.e., fish will appear at a distance 5.3 cm from E towards F (less than actual distance, i.e., 6 cm)

$$(b) \quad \text{Seen from F } \mu_1 = \frac{4}{3}, \quad \mu_2 = 1, \quad R = -10 \text{ cm and } u = -(10 + 4) = -14 \text{ cm}$$

$$\Rightarrow \frac{1}{v} - \frac{\frac{4}{3}}{-14} = \frac{1 - \frac{4}{3}}{-10} \Rightarrow v = \frac{-210}{13} = -16.154 \text{ cm}$$

so fish will appear at a distance 16.154 cm from F toward E (more than actual distance, i.e., 14 cm)

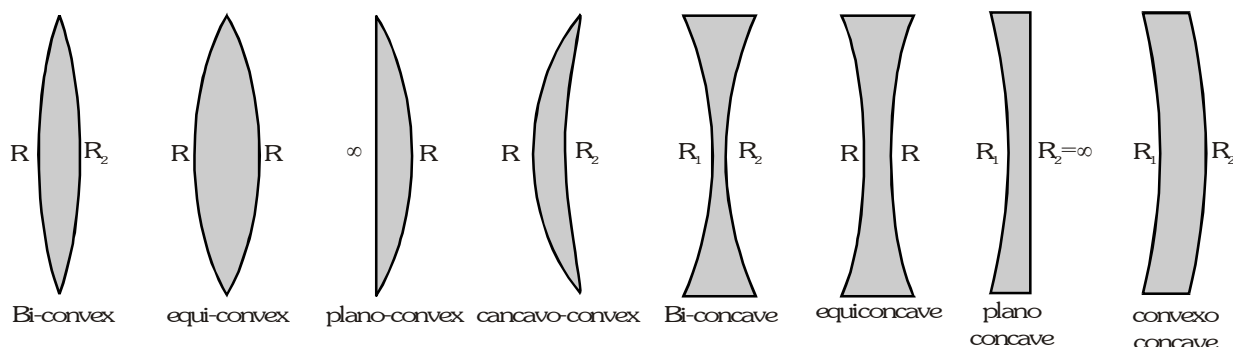


LENS

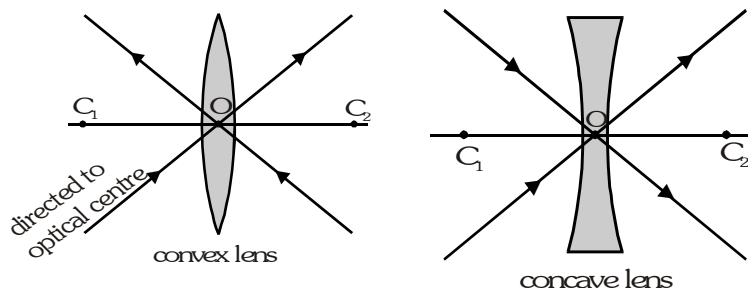
A lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e. paraxial) portion is thicker than marginal one.

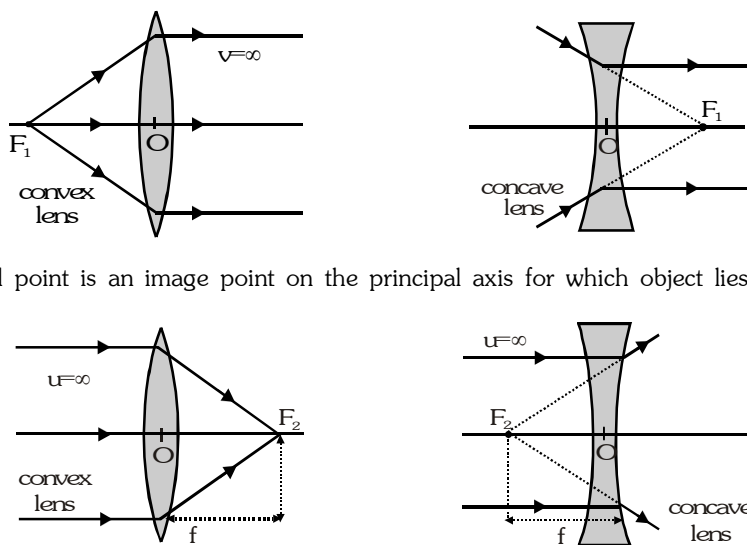
However if the central portion of a lens is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens. This is how we can identify convergent and divergent lenses.



- ⊙ **Optical Centre :** O is a point for a given lens through which any ray passes undeviated



- ⊙ **Principal Axis :** $C_1 C_2$ is a line passing through optical centre and perpendicular to the lens.
 ⊙ **Principal Focus :** A lens has two surfaces and hence two focal points. First focal point is an object point on the principal axis for which image is formed at infinity.



While second focal point is an image point on the principal axis for which object lies at infinity

- ⊙ **Focal Length f** is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge.
 ⊙ **Aperture :** In reference to a lens, aperture means the effective diameter. Intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e.,
 Intensity \propto (Aperture)²

LENS-MAKER'S FORMULA

In case of image formation by a lens

Image formed by first surface acts as object for the second.

So, from the formula of refraction at curved surface. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

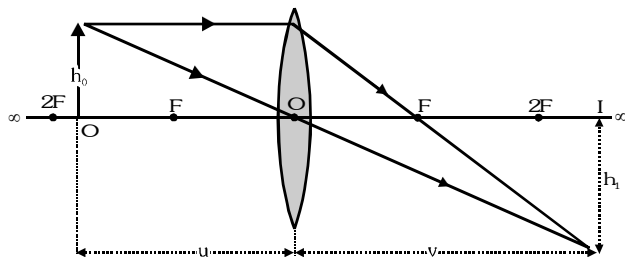
For first surface A $\frac{\mu_L}{v_1} - \frac{\mu_M}{u} = \frac{\mu_L - \mu_M}{R_1}$... (i) $[\because \mu_2 = \mu_L, \mu_1 = \mu_M]$

For second surface B $\frac{\mu_M}{v} - \frac{\mu_L}{v_1} = \frac{\mu_M - \mu_L}{R_2} = -\frac{\mu_L - \mu_M}{R_2}$... (ii) $[\because \mu_2 = \mu_M, \mu_1 = \mu_L, \mu_1 = \mu_2, u \rightarrow v_1]$

By adding (i) and (ii)

$$\mu_M \left[\frac{1}{v} - \frac{1}{u} \right] = (\mu_L - \mu_M) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{\mu_L - \mu_M}{\mu_M} \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \dots (iii) \quad \left(\because \mu = \frac{\mu_L}{\mu_M} \right)$$

Now if object is at infinity, Image will be formed at the focus, $u = -\infty, v = f$ So $\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \dots (iv)$



This is known as lens maker's formula by equating (iii) and (iv) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ this is known as lens formula

Magnification : $m = \frac{\text{height of image}}{\text{height of object}} = \frac{h_1}{h_0} = \frac{v}{u} = \frac{f}{f + u} = \frac{f - v}{f}$

RULES FOR IMAGE FORMATION

- ⊙ A ray passing through optical centre proceeds undeviated through the lens
- ⊙ A ray passing through first focus or directed towards it, after refraction from the lens, becomes parallel to the principal axis.
- ⊙ A ray passing parallel to the principal axis after refraction through the lens passes or appears to pass through F_2

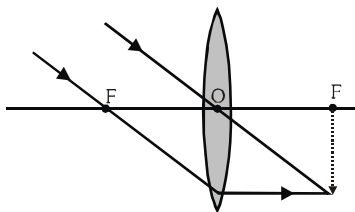
For Convergent or convex lens

Object	Image	Magnification
$-\infty$	F	$ m \ll 1$ & $m < 0$
$-\infty - 2F$	F-2F	$ m < 1$ & $m < 0$
2F	2F	$m = -1$
F - 2F	$\infty - 2F$	$ m > 1$ & $m < 0$
Just before F towards C	$+\infty$	$m \ll -1$
Just before F towards P	$-\infty$	$m \gg 1$
F - O	In front of lens	$m > 1$

IMAGE FORMATION FOR CONVEX LENS (CONVERGENT LENS)

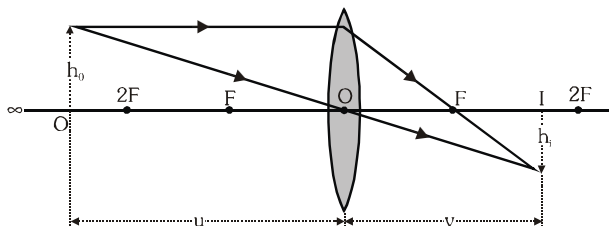
- (i) Object is placed at infinity

Image :
at F real inverted very small in size
 $|m| \ll 1$ & $m < 0$



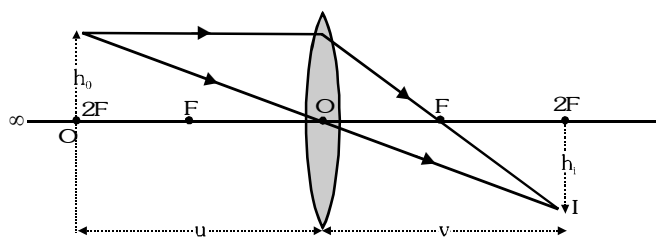
- (ii) Object is placed in between $\infty - 2F$

Image :
real (F - 2F) inverted small in size (diminished)
 $|m| < 1$ & $m < 0$



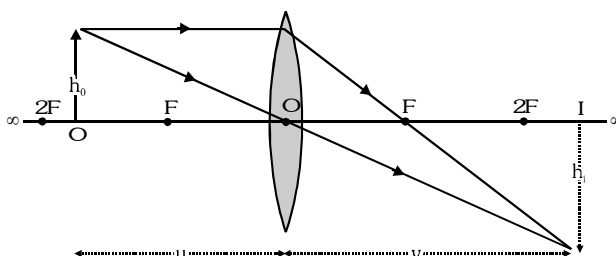
- (iii) Object is placed at 2F

Image :
real (at 2F) inverted equal (of same size)
($m = -1$)



- (iv) Object is placed in between 2F - F

Image :
real (2F - ∞) inverted enlarged
 $|m| > 1$ & $m < 0$



- (v) Object is placed in between F - O

Image :
virtual (in front of lens) erected enlarge
($m > +1$)

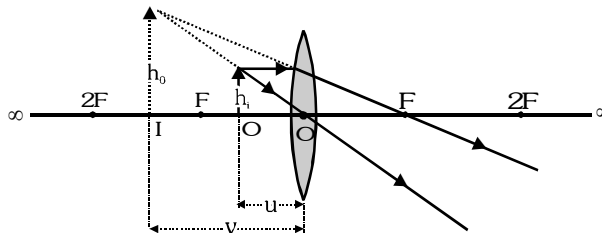
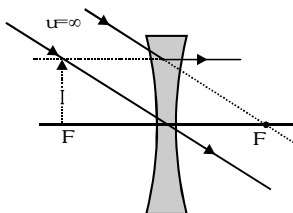


IMAGE FORMATION FOR CONCAVE LENS (DIVERGENT LENS)

Image is virtual, diminished, erect, towards the object, $m = +ve$

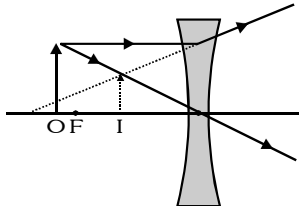
- (i) Object is placed at infinity

Image :
At F virtual erected
diminished ($m \ll +1$)



- (ii) Object is placed in front of lens

Image :
between F and optical centre
virtual erected diminished ($m < +1$)



Sign convention for object/image for lens

Real object	$u - ve$
Real image	$v + ve$
Virtual object	$u + ve$
Virtual image	$v - ve$

POWER OF LENS

Reciprocal of focal length in meter is known as power of lens.

SI UNIT : dioptre (D) **Power of lens** : $P = \frac{1}{f(m)} = \frac{100}{f(cm)}$ dioptre [in air]

COMBINATION OF LENSES

Two thin lens are placed in contact to each other

power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention when solve numericals

Two thin lens are placed in at a small distance d

(provided incident rays are parallel to principal axis).

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow P = P_1 + P_2 - d P_1 P_2$$

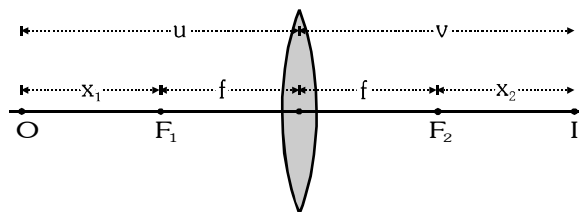
Use sign convention when solving numericals

© Newton's Formula

$$f = \sqrt{x_1 x_2}$$

x_1 = distance of object from focus.

x_2 = distance of image from focus.



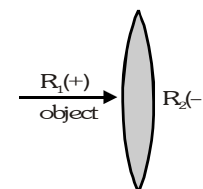
SOME SPECIAL CASES

(i) The focal length of equiconvex lens placed in air

refractive index of lens $\mu_L = \mu$ refractive index of medium $\mu_M = 1$

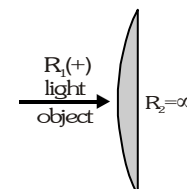
$$R_1 = +R, \quad R_2 = -R$$

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R} - \left(-\frac{1}{R} \right) \right] \Rightarrow \text{Focal length } f = \frac{R}{2(\mu - 1)}$$



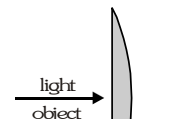
(ii) Focal length of planoconvex lens placed in air

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R} - \frac{1}{\infty} \right] \Rightarrow \text{Focal length } f = \frac{R}{(\mu - 1)}$$



If object is placed towards plane surface

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{\infty} - \left(-\frac{1}{R} \right) \right] \Rightarrow \text{Focal length } f = \frac{R}{(\mu - 1)}$$



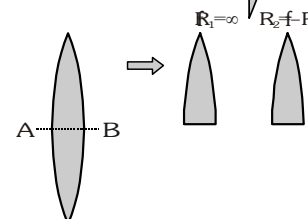
(iii) If an equiconvex lens of focal length f is cut into equal parts by a horizontal

plane AB then the focal length of each part will be equal to that of initial lens.

Because μ , R_1 and R_2 will remain unchanged. Only intensity will be reduced.

\therefore intensity $I \propto (\text{apertures})^2$

\therefore intensity through a single part will be reduced

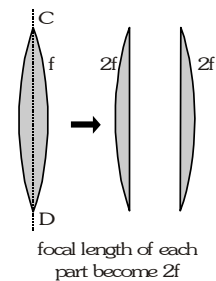


- (iv) If the same lens is cut into equal parts by a vertical plane CD the focal length of each part will be double of initial value but intensity will remain unchanged.

For equiconvex lens $\frac{1}{f} = \frac{(\mu-1)2}{R}$ For plano convex lens $\frac{1}{f_1} = \frac{\mu-1}{R}$

So $\frac{1}{f} = \frac{2}{f_1} \Rightarrow f_1 = 2f \Rightarrow$ Focal length of each part = 2

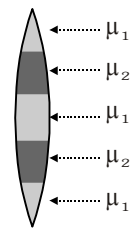
(focal length of original lens)



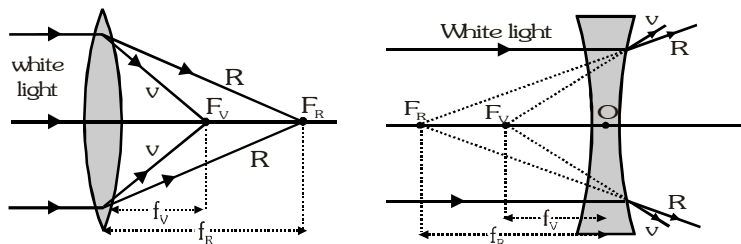
- (v) If a lens is made of number of layers of different refractive index for a given wavelength

then no. of images is equal to number of refractive index, as $\frac{1}{f} \propto (\mu-1)$

In figure number of images = 2

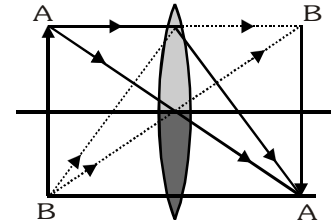


- (vi) Focal length of lens depends on wavelength. $\therefore \frac{1}{f} \propto (\mu-1) \propto \frac{1}{\lambda} [f \propto \lambda]$ $f_R > f_V$



- (vii) If half portion of lens is covered by black paper then intensity of image

will be reduced but complete image will be formed.

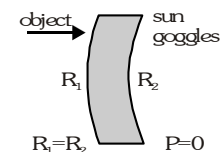


- (viii) Sun-goggles :

radius of curvature of two surfaces is equal with centre on the same side

$R_1 = R_2 = +R$ so $\frac{1}{f} = (\mu-1) \left[\frac{1}{R} - \frac{1}{R} \right]$

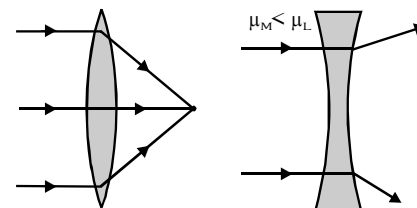
$\Rightarrow \frac{1}{f} = 0 \Rightarrow f = \infty$ and $P = 0 \Rightarrow$ sun goggles have no power



- (ix) If refractive index of medium < Refractive index of lens

If $\mu_M < \mu_L$ then $f = +ve$

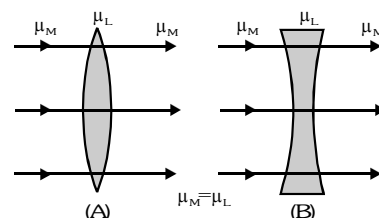
Convex lens behave as convex lens. While concave lens behave as concave lens.



- (x) Refractive index of medium = Refractive index of lens ($\mu_M = \mu_L$)

$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$; $\frac{1}{f} = 0 \Rightarrow f = \infty$ & $P = 0$

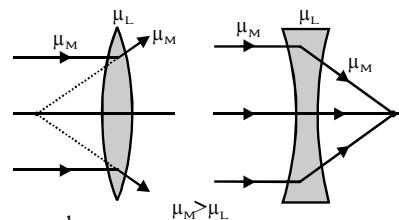
Lens will behave as plane glass plate



(xi) Refractive index of medium > Refractive index of lens

$$\mu_M > \mu_L \Rightarrow \frac{\mu_L}{\mu_M} < 1 \quad [f \text{ will be negative}]$$

convex lens will behave as concave lens and concave lens will behave as convex lens. If a air bubble is formed in water it behaves as concave lens.



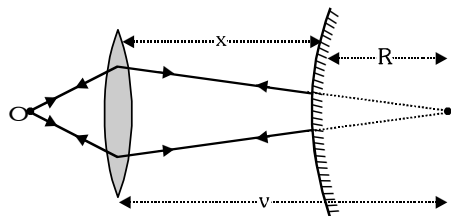
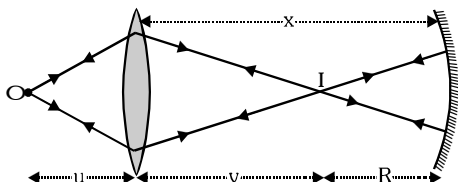
Example

A point source S is placed at distance of 15 cm from a converging lens of focal length 10 cm. Where should a (i) concave mirror (ii) convex mirror of focal length 12 cm be placed so that real image is formed on object itself.

Solution

$$u = -15\text{cm}, f = +10\text{cm}; \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-15)} = \frac{1}{10} \Rightarrow v = 30\text{ cm}$$

$$(i) x = v + 2f \Rightarrow 30 + 2 \times 12 = 54\text{ cm} \quad (ii) x = v - 2f = 30 - 2 \times 12 = 6\text{ cm}$$



Example

A convex lens of focal length f is producing real image which is $\frac{1}{n}$ times of the size of the object. Find out position of the object.

Solution

$$\text{Image is real so } m = \frac{v}{u} = -\frac{1}{n} \Rightarrow v = -\frac{u}{n}$$

$$\text{from lens formula } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-\frac{u}{n}} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{-(n+1)}{u} = \frac{1}{f} \Rightarrow u = -f(1+n)$$

Example

- If $f = +0.5\text{m}$, what is the power of the lens ?
- The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass ?
- A convex lens has 20 cm focal length in air. What is the focal length in water ? (Refractive index of air-water = 1.33, refractive index for air glass is 1.5)

Solution

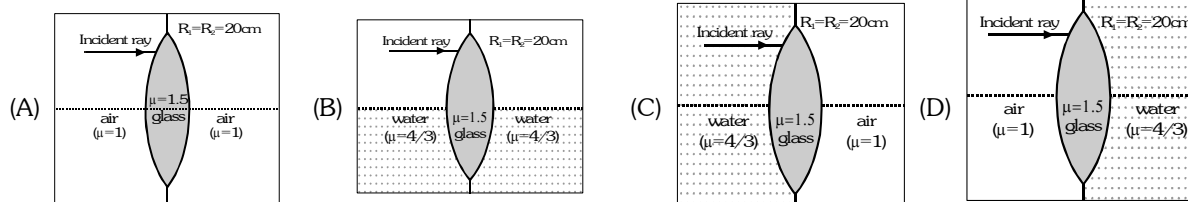
$$(a) \quad P = \frac{1}{f(\text{m})} = \frac{1}{0.5} = +2\text{D}$$

$$(b) \quad \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \Rightarrow \frac{1}{12} = (\mu - 1) \left(\frac{1}{10} - \frac{1}{-15} \right) \Rightarrow \mu = 1.5$$

$$(c) \quad \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \text{ so } \frac{1}{f} \propto (\mu - 1); \quad \frac{f_w}{f_a} = \frac{(\mu_\ell - 1)}{(\mu_w - 1)} \Rightarrow f_w = \frac{(1.5 - 1)}{(1.33 - 1)} \times 20 = 78.2\text{cm}$$

Example

Column I (optical system)



Column II (focal length)

(P) 80 cm

(Q) 40 cm

(R) 30 cm

(S) 20 cm

Solution

Ans. (A) -S (B) -P (C) -R

$$\text{For (A) : } \frac{1}{f} = (\mu - 1) \left(\frac{1}{p_1} + \frac{1}{p_2} \right) = (1.5 - 1) \left(\frac{1}{20} + \frac{1}{20} \right) = \frac{1}{20} \Rightarrow f = 20 \text{ cm}$$

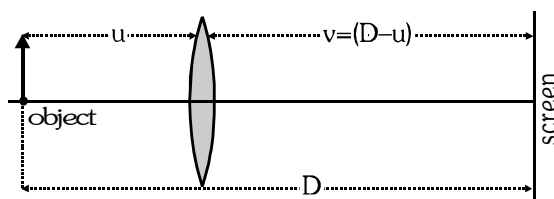
$$\text{For (B) : } \frac{1}{f} = \left(\frac{1.5}{4/3} - 1 \right) \left(\frac{1}{20} + \frac{1}{20} \right) = \frac{1}{80} \Rightarrow f = 80 \text{ cm}$$

$$\text{For (C) : } \frac{1.5}{v_1} - \frac{4/3}{\infty} = \left(1.5 - \frac{4}{3} \right) \left(\frac{1}{20} \right) \quad \& \quad \frac{1}{f} - \frac{1.5}{v_1} = (1 - 1.5) \left(\frac{1}{-20} \right) \Rightarrow f = 30 \text{ cm}$$

$$\text{For (D) : } \frac{1.5}{v_1} - \frac{1}{\infty} = (1.5 - 1) \left(\frac{1}{20} \right) \quad \& \quad \frac{4/3}{f} - \frac{1.5}{v_1} = \left(\frac{4}{3} - 1.5 \right) \left(-\frac{1}{20} \right) \Rightarrow f = 40 \text{ cm}$$

DISPLACEMENT METHOD

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart. If $D > 4f$ there are two positions of lens at which a sharp image of the object is formed on the screen



By lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{D-u} - \frac{1}{-u} = \frac{1}{f} \Rightarrow u^2 - Du + Df = 0 \Rightarrow u = \frac{D \pm \sqrt{D(D-4f)}}{2}$ there are three possibilities

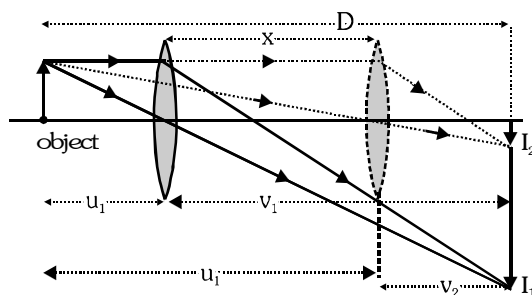
(i) for $D < 4f$ u will be imaginary hence physically no position of lens is possible

(ii) for $D = 4f$ $u = \frac{D}{2} = 2f$ so only one position of lens is possible

and since $v = D - u = 4f - 2f = u = 2f$

(iii) for $D > 4f$ $u_1 = \frac{D - \sqrt{D(D-4f)}}{2}$ and $u_2 = \frac{D + \sqrt{D(D-4f)}}{2}$

So there are two positions of lens for which real image will be formed on the screen. (for two distances u_1 and u_2 of the object from lens)



If the distance between two positions of lens is x then

$$x = u_2 - u_1 = \frac{D + \sqrt{D(D-4f)}}{2} - \frac{D - \sqrt{D(D-4f)}}{2} = \sqrt{D(D-4f)} \Rightarrow x^2 = D^2 - 4Df \Rightarrow f = \frac{D^2 - x^2}{4D}$$

Distance of image corresponds to two positions of the lens :

$$v_1 = D - u_1 = D - \frac{1}{2}[D - \sqrt{D(D-4f)}] = \frac{1}{2}[D + \sqrt{D(D-4f)}] = u_2 \Rightarrow v_1 = u_2$$

$$v_2 = D - u_2 = D - \frac{1}{2}[D + \sqrt{D(D-4f)}] = \frac{1}{2}[D - \sqrt{D(D-4f)}] = u_1 \Rightarrow v_2 = u_1$$

for two positions of the lens distances of object and image are interchangeable.

Now $x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + u_1$ [$\because v_1 = u_2$]

$$\text{so } u_1 (= v_2) = \frac{D-x}{2} \quad \text{and} \quad v_1 = \frac{D+x}{2} = u_2; \quad m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \frac{D+x}{D-x} \quad \text{and} \quad m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \frac{D-x}{D+x}$$

$$\text{Now } m_1 \times m_2 = \frac{D+x}{D-x} \times \frac{D-x}{D+x} \Rightarrow \frac{I_1 I_2}{O^2} = 1 \Rightarrow O = \sqrt{I_1 I_2}$$

Example

A convex lens is placed between an object and a screen which are at a fixed distance apart for one position of the lens. The magnification of the image obtained on the screen is m_1 . When the lens is moved by a distance d the magnification of the image obtained on the same screen is m_2 . Find the focal length of the lens.

Solution

If D is the distance between the object and the screen, d the separation of the two position of lens throwing two images on the screen then

$$m_1 = \frac{(D+d)}{(D-d)} \quad \text{and} \quad m_2 = \frac{(D-d)}{(D+d)} \quad \therefore m_1 - m_2 = \frac{4Dd}{D^2 - d^2} \quad \text{but} \quad \frac{D^2 - d^2}{4D} = f \quad \text{so} \quad m_1 - m_2 = \frac{d}{f} \Rightarrow f = \frac{d}{m_1 - m_2}$$

Example

In a displacement method using lens, we obtain two images for separation of the lens d . One image is magnified as much as the other is diminished. If m is the magnifications of one image, find the focal length of the lens.

Solution

$$\text{From above question } f = \frac{d}{m_1 - m_2} \quad \text{here if } m_1 \text{ is taken as } m, \quad m_2 = \frac{1}{m}, \quad \text{so } f \text{ becomes } \frac{md}{(m^2 - 1)}$$

Example

In the displacement method the distance between the object and the screen is 70 cm and the focal length of the lens is 16 cm, find the separations of the magnified and diminished image position of the lens.

Solution

$$d = \sqrt{D^2 - 4fd} = \sqrt{(70)^2 - 4 \times 16 \times 70} = \sqrt{420} = 20.5 \text{ cm}$$

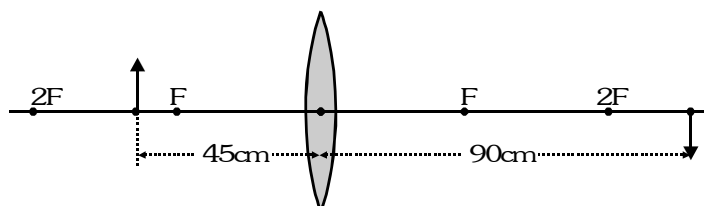
Example

An object 25 cm high is placed in front of a convex lens of focal length 30 cm. If the height of image formed is 50 cm, find the distance between the object and the image (real and virtual) ?

Solution

As object is in front of the lens, it is real and as $h_1 = 25$ cm, $f = 30$ cm, $h_2 = -50$ cm ; $m = \frac{h_2}{h_1} = \frac{-50}{25} = -2$

$$m = \frac{f}{f+u} \Rightarrow -2 = \frac{30}{30+u} \Rightarrow u = -45 \text{ cm} \Rightarrow m = \frac{v}{u} \Rightarrow -2 = \frac{v}{-45} \Rightarrow v = 90 \text{ cm}$$



As in this situation object and image are on opposite sides of lens, the distance between object and image $d_1 = u + v = 45 + 90 = 135$ cm. If the image is erect (i.e., virtual)

$$m = \frac{f}{f+u} \Rightarrow 2 = \frac{30}{30+u} \Rightarrow u = -15 \text{ cm} \Rightarrow m = -\frac{v}{u} \Rightarrow 2 = \frac{-v}{-15} \Rightarrow v = 30 \text{ cm}$$

As in the situation both image and object are in front of the lens, the distance between object and image $d_2 = v - u = 30 - 15 = 15$ cm.

COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used, the image formation is considered one after another in steps, The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens acts as an object for the third, and so on, The total magnification in such situations will be given by

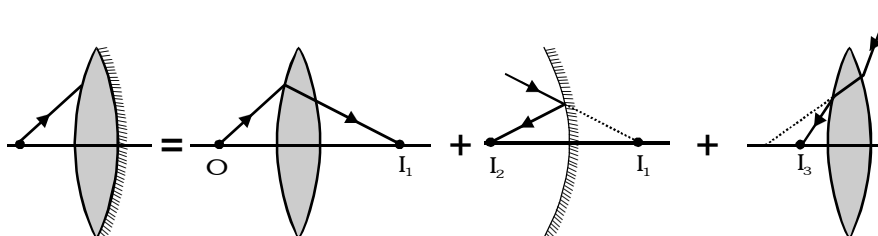
$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \dots \Rightarrow m = m_1 \cdot m_2 \cdot \dots$$

Power of Lens [in air] $P_L = \frac{1}{f_L}$ Converging lens $P_L = +ve$ Diverging lens $P_L = -ve$

Power For mirror $P_M = -\frac{1}{f_m}$ Convex mirror $P_M = -ve$ Concave mirror $P_M = +ve$

SILVERING OF LENS

Calculate equivalent focal length of a equiconvex lens silvered at one side.



$$P = P_L + P_M + P_L = 2P_L + P_M$$

$$\frac{1}{F} = \frac{1}{f_L} + \frac{1}{f_m} + \frac{1}{f_L} = \frac{2}{f_L} + \frac{1}{f_m} = \frac{2(\mu-1) \times 2}{R} + \frac{2}{R} = \frac{4\mu-4+2}{R} \Rightarrow F = \frac{R}{4\mu-2}$$

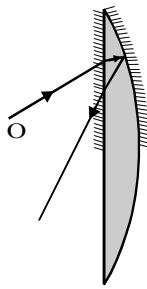
Example

Calculate equivalent focal length of plano convex lens for following case :-

- When curved surface is silvered.
- When plane surface is silvered.

Solution

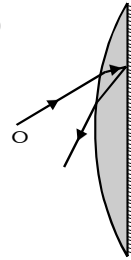
(i) $P = 2P_L + P_M$



$$\Rightarrow \frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m} \Rightarrow \frac{1}{F} = \frac{2(\mu - 1)}{R} + \frac{2}{R}$$

$$\Rightarrow \frac{1}{F} = \frac{2\mu}{R} \Rightarrow F = \frac{R}{2\mu}$$

(ii) $P = 2P_L + P_M$



$$\Rightarrow \frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m} \Rightarrow \frac{1}{F} = \frac{2(\mu - 1)}{R} + \frac{1}{\infty}$$

$$\Rightarrow F = \frac{R}{2(\mu - 1)}$$

Example

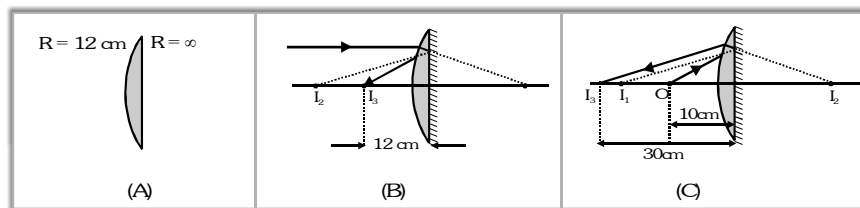
The radius of curvature of the convex face of a plano-convex lens is 12 cm and its refractive index is 1.5.

- Find the focal length of this lens. The plane surface of the lens is now silvered.
- At what distance from the lens will parallel rays incident on the convex face converge ?
- Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.
- Calculate the image distance when the object is placed as in (c).

Solution

- As for a lens, by lens-maker's formula $\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$ Here $\mu = 1.5$; $R_1 = 12$ cm and $R_2 = \infty$

So $\frac{1}{f} = (1.5 - 1) \left[\frac{1}{12} - \frac{1}{\infty} \right]$ i.e. $f = 24$ cm i.e., the lens as convergent with focal length 24 cm.



- As light after passing through the lens will be incident on the mirror which will reflect it back through the lens

again, so $P = P_L + P_M + P_L = 2P_L + P_M$ But $P_L = \frac{1}{f_L} = \frac{1}{0.24}$ and $P_M = -\frac{1}{\infty} = 0$ $\left[\text{as } f_M = \frac{R}{2} = \infty \right]$

So $P = 2 \times \frac{1}{0.24} + 0 = \frac{1}{0.12}$ D. The system is equivalent to a concave mirror of focal length F, $P = -\frac{1}{F}$

i.e., $F = -\frac{1}{P} = -0.12$ m = -12 cm i.e., the rays will behave as a concave mirror of focal length 12 cm.

So as for parallel incident rays $u = -\infty$, from mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{v} + \frac{1}{-\infty} = \frac{1}{-12}$

$\Rightarrow v = -12$ cm i.e., parallel incident rays will focus at a distance of 12 cm in front of the lens as shown in Figure (c) and (d)

When object is at 20 cm in front of the given silvered lens which behaves as a concave mirror of focal length 12 cm, from mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{v} + \frac{1}{-20} = \frac{1}{-12}$

$\Rightarrow v = -30$ cm i.e., the silvered lens will form image at a distance of 30 cm in front of it as shown in fig. (C)

Example

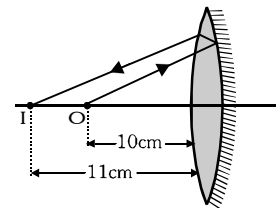
A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual ?

Solution

As radius of curvature of silvered surface is 22 cm,

$$\text{so } f_M = \frac{R}{2} = \frac{-22}{2} = -11 \text{ cm} = -0.11 \text{ m}$$

$$\text{and hence, } M = -\frac{1}{f_M} = -\frac{1}{-0.11} = \frac{1}{0.11} \text{ D}$$



Further as the focal length of lens is 20 cm, i.e., 0.20 m its power will be given by : $P_L = \frac{1}{f_L} = \frac{1}{0.20} \text{ D}$.

Now as in image formation, light after passing through the lens will be reflected back by the curved mirror

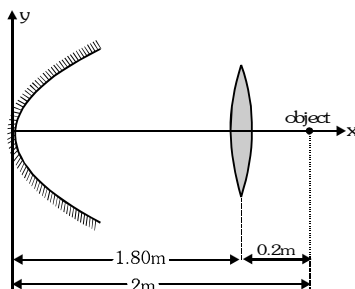
through the lens again $P = P_L + P_M + P_L = 2P_L + P_M$ i.e. $P = \frac{2}{0.20} + \frac{1}{0.11} = \frac{210}{11} \text{ D}$.

So the focal length of equivalent mirror $F = -\frac{1}{P} = -\frac{11}{210} \text{ m} = -\frac{110}{21} \text{ cm}$ i.e., the silvered lens behave as a concave

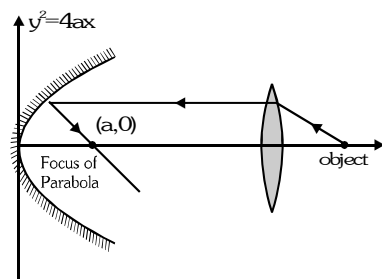
mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it, $\frac{1}{v} + \frac{1}{-10} = -\frac{21}{110}$ i.e., $v = -11 \text{ cm}$ i.e., image will be 11 cm in front of the silvered lens and will be real as shown in Figure.

Example

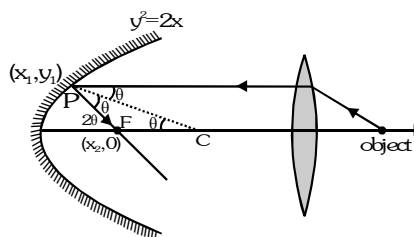
A point object is kept at a distance of 2m from a parabolic reflecting surface $y^2 = 2x$. An equiconvex lens is kept at a distance of 1.80 m from the parabolic surface. The focal length of the lens is 20 cm. Find the position from origin of the image in cm, after reflection from the surface.



Solution



OR



Comparing with $y^2 = 4ax \Rightarrow a = 0.5$

PC is a normal so $\tan(\pi - \theta) = \frac{-1}{(dy/dx)_{x_1, y_1}} = -y_1 \Rightarrow$ final position of image = 0.5 m = 50 cm

$$\text{But } \tan 2\theta = \frac{y_1 - 0}{x_2 - x_1} \text{ \& \ } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \Rightarrow \frac{y_1}{x_2 - x_1} = \frac{2(y_1)}{1 - y_1^2} x_2 = \frac{1}{2} \text{ m}$$